10

PRACTICAL TRADE MATHEMATICS

FOR ELECTRICIANS, MACHINISTS, CARPENTERS, PLUMBERS, AND OTHERS

BY

JAMES A. MOYER

Formerly in charge of Division of Engineering Calculations in General Electric Company

AND

CHARLES H. SAMPSON, B. S.

Head of Technical and Mathematical Departments, Huntington School, Boston, Mass.

NEW YORK

JOHN WILEY & SONS, INC.

LONDON: CHAPMAN & HALL, LIMITED

1920

ATLANTIC PRINTING COMPANY
CAMBRIDGE, MASS., U. S. A.

PREFACE

Increasing demands for instruction in practical or trade subjects have made it necessary to develop courses of study having special application to particular trades. In the past much of this instruction has been done without textbooks, or if textbooks have been used, they have not been satisfactory. Since, in many cases, it has been necessary to supplement the class-room instruction with lesson material prepared by some method of duplication other than printing, it has been brought to the attention of publishers that here is an undeveloped field, particularly for books in mathematics. Old-fashioned academic books are not at all satisfactory, for example, in the teaching of Practical Mathematics to classes of electricians and machinists. These trades have many problems, more or less related, which are concrete examples of the day's work and can be effectively used for study material. These chapters have been prepared with the object, first, of establishing and holding the interest of adult students; and second, of presenting for study only those parts of the common mathematical subjects which a qualified electrician or machinist will be likely to use either in his present employment or in the future work he will do as a result of study and advancement.

Throughout this book, as much as possible, the language used by practical men has been followed, and the presentation has been made direct, intimate, and personal. Whenever possible, unusual mathematical terms, symbols, and names have been avoided. For example, in nearly all parts of this book names like "digit," "factor," "multiplicand," etc., have been avoided and the mathematical ideas usually

conveyed by these terms have been stated in another way. Adult students who have been out of school for a number of years and have come to realize their need of more education will certainly appreciate the avoidance of mathematically technical terms.

Much attention has been given, particularly as regards the common mathematical operations, to the importance of *checking calculations*. An estimating or designing electrician or machinist who cannot be sure of his figures and his calculations works under a great handicap. A mistake may spoil all his chances of success. Therefore, the importance of learning to check calculations should be obvious. In addition to the usual methods of checking there is a chapter on graphic methods of representing numerical data. It deals mostly with plotting curves on cross-section paper. Such plotting of curves is one of the most effective methods of eliminating errors. If, after applying checks, the numerical results secured by various methods are found to agree, an electrician, machinist, or engineer is able to tell with definiteness and assurance whether or not his calculations are correct. The authors know of no better way to test the ability of men in practical work than by ascertaining their ability to check their calculations accurately.

Two groups of problems are given at the end of each chapter. The first group is intended to include general applications of mathematics, especially for machinists, draftsmen, plumbers, loom fixers, carpenters, steam engineers, firemen and sheet-metal workers. The second group is especially for students engaged in electrical trades whose interests are as a rule somewhat more specialized than the other trades mentioned. Teachers will find, however, that some classes with general trade representation will be more interested in the electrical group than in the general group of problems, and problem assignments should be modified accordingly.

These groups of problems represent years of painstaking effort and will doubtless be appreciated by teachers in trade schools and by teachers of adult trade classes. In collecting these problems efforts were made to include those that would be of definite, practical interest and benefit to men engaged in the trades mentioned.

A number of teachers were from time to time associated with the authors in the preparation of this book. Special mention should be made of Charles W. Hobbs and Herbert A. Dallas, of the Division of University Extension, Massachusetts Department of Education; William E. McClintock, formerly chief engineer of the Massachusetts Highway Commission; Professor L. W. Hitchcock, of New Hampshire State College; Joseph W. L. Hale, Federal Board for Vocational Education; and John H. Buck, of Rindge Manual Training School, Cambridge, Massachusetts. Miss Elizabeth McCausland has contributed valuable work in the preparation of figures. The courtesy of the General Electric Company is acknowledged for helpful contributions.

THE AUTHORS.

BOSTON, MASSACHUSETTS,
September, 1920.

CONTENTS

SYMBOLS FOR PRACTICAL MATHEMATICS

c = cutting speed in feet per minute
C = electric current in amperes
cts. or ¢ = cent or cents
d = diameter of smaller gear or pulley
D = diameter of larger gear or pulley
e = pull on belts in pounds
E = voltage (e.m.f.) in volts
ft. or ′ = foot or feet
h.p. = horse power
in. or ″ = inch or inches
kw. = electric power in kilowatts (1000 watts)
lb. = pound or pounds
l.c.d. = lowest (smallest) common denominator
M. = thousand
M.B.F. = thousand board feet
n = revolutions per minute of smaller gear or pulley
N = revolutions per minute of larger gear or pulley, also number of wires
r = revolutions per minute (in cutting speed formulas)
R = resistance in ohms
r.p.m. = revolutions per minute
V = volts
v = velocity in feet per minute
W = weight in pounds, also electric power in watts
x, y, and z = unknown quantities
π or Pi = 3.1416 or $\frac{22}{7}$ (pronounced like *pie*)

Practical Trade Mathematics

CHAPTER I

THE USE OF NUMBERS. SIMPLE MEASURES

Addition. Adding, or finding the sum of two or more numbers, is probably the commonest application of mathematics. We are all familiar with the adding of money. For example, if we have purchased in a store an article costing $6.00 and an article costing $8.00, we find by the addition of $6.00 and $8.00 that the total cost is $14.00.

Remember that only quantities of the same kind can be added. Thus we cannot add dollars and doughnuts or dollars and cents together by merely adding the numbers. For example, if we should add 120 dollars and 503 cents, we should have the quantity 623 as the sum of 120 and 503, but it would be neither dollars nor cents. Similarly, we cannot add yards to ounces nor nails to screws.

All numbers, however large, are a collection of figures (or digits). Thus, the number 623 consists of three figures, 6, 2 and 3. The figure 3, farthest to the right, is the smallest in value and represents *units*. As we go from the right to the left, each figure represents a larger value. The names of the figures or digits in a number of ten figures are given below.

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units
5,	7	8	6,	4	8	3,	6	5	7

This number will be read five billion, seven hundred eighty-six million, four hundred eighty-three thousand, six hundred fifty-seven.

Numbers to be added must be placed so that their corresponding figures will be in vertical columns, putting all the units from each of the numbers to be added in the *units column*, all the tens in the *tens column*, all the hundreds in the *hundreds column*, and so on. This can be illustrated by the following example to find the total number of feet in the following four lines of piping:

6	feet
45	"
472	"
1275	"
1798	"

The units column (farthest to the right) when added amounts to eighteen, which is equivalent to "one ten and eight left over." In going through with this addition we put the eight in the units column and add the ten to the tens column. Continuing this method as we add each column, we obtain by addition 1798 feet, which is the total number of feet of piping in the four lines.

Subtraction. Finding the difference between two numbers is subtraction. It is the reverse of addition. In subtracting, remember that the smaller number or quantity must be subtracted from the larger. If, for example, a box contains 12 files, it is obviously impossible actually to subtract, or take away, more than 12 files. We can, however, take away 7 files from 12 files and have remaining 5 files.

Signs or Symbols. We often use the following signs or symbols for the purpose of saving time in writing:

+ (read "plus") means addition
− (read "minus") " subtraction
= (read "equals") " equal to

These signs, or symbols, are used in mathematics for the same reason that a stenographer uses outlines or signs in writing letters and similar work requiring speed. They are short cuts. With the help of these symbols we could write the last example in *addition* as follows: 6 feet + 45 feet + 472 feet + 1275 feet = 1798 feet. Similarly, the example in subtraction stated above could be written 12 files − 7 files = 5 files.

Note that in each example the kind of quantity to be added or subtracted is the same. We cannot add feet to files, if we want a total in files. Many errors in mathematics may be avoided if you adopt the plan of writing always the *name of the kind of quantity*, for example, 7 feet, 12 barrels, 24 bolts. It happens frequently that calculations must be made involving pressures, as for example, boiler pressures. For some calculations of this kind, particularly for the strength of a boiler, the pressures are ordinarily stated in pounds per square inch, while if the calculations are for finding out how much horse power is obtainable from steam in a new design of steam engine, pressures will be most probably used in pounds per square foot. For this and similar reasons, you cannot be too careful in stating the kinds of quantities or, in other words, the kinds of units employed in calculations.

Multiplication is in reality a short cut in addition. Fig. 1 illustrates a package of carpenters' pencils packed in

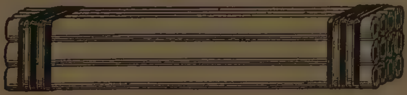


FIG. 1. Example in Multiplication.

three horizontal rows with four pencils in each row. One way to find the total number of pencils in the package is to

count the individual pencils, row after row. Another way is as follows:

> There are 4 pencils in each row.
> There are 3 rows.
> As there are 4 pencils in each row and there are 3 rows, the total number of pencils equals 3 times 4 or twelve pencils.

The sign or symbol to show multiplication is $\times$, which should be read "times" or "multiplied by." The last example would then be expressed by signs or symbols as follows:

$$3 \times 4 \text{ pencils} = 12 \text{ pencils.}$$

Division is finding how many times one number is contained in another number. It is the reverse of multiplication. If we want to find out how many iron bars, each 5 inches long, can be cut from a bar 40 inches long, we can, of course, find out by making one sample bar 5 inches long and, using this as a measure, lay off or mark along the bar the points where the bar should be cut. The number of 5-inch pieces or divisions obtainable may be then counted. The count shows that we shall have 8 pieces.

In the short-cut calculation which we call division, we find out how many times the smaller number, which is called the *divisor*, is contained in the larger number. Thus, a 40-inch piece divided by a 5-inch piece gives 8 pieces. This may be expressed by the use of the sign, or symbol, for division ($\div$) as:

$$40 \div 5 = 8.$$

You should learn *not to depend* on given answers. Most answers can be checked, and it is so important to get the correct result, *especially in practical work*, that you should learn to become independent and check your results as far

as possible by your own reasoning and understanding of the problem.

Checking in Addition. A good way to check the accuracy of a result in addition is to put down, *below* the numbers included in the addition (see Check A below), the sum of each column of figures separately arranged and added as indicated. Be sure to set each total one place to the left of the preceding one, as shown in the example.

Problem in Addition	Check A	Check B
423	423	1324
71	71	423
8	8	71
795	795	8
27	27	795
1324	24	27
	20	*1324*
	11	
	1324	

The sum of the first right-hand column is 24. The sum of the second column is 20. The sum of the third column is 11. Note that the figures in italics, *1324*, check with the answer of the problem. Remember that no check is "fool proof." "Use your head" in both solution and check.

Another method of checking addition is first to add upward, and in checking add downward. Arrange as in Check B above.

Checking in Subtraction. To the difference obtained by subtracting add the number which was subtracted, thus,

$$84 - 23 = 61$$
$$\text{Check: } 61 + 23 = 84$$

Checking in Multiplication. *Method* 1. — Reverse the work (interchange the numbers) and multiply.

```
Problem          Check
   273              49
    49             273
  ----             ---
  2457             147
 1092             343
 -----            98
 13377           -----
                 13377
```

Method 2. — Divide the number obtained from multiplication by either of the two numbers originally multiplied together. The division by one of these numbers will give the other. See Check A or Check B below.

```
Problem        Check A            Check B
   273              273                  49
    49        49)13377           273)13377
  ----            98                 1092
  2457            ---                ----
 1092             357                 2457
 -----            343                 2457
 13377            ---                 ----
                   147
                   147
                   ---
```

The position of the answer (sometimes called the quotient) in Check A and Check B as well as in later examples of division is recommended because it saves space and, as will be seen later, makes easier the placing of the "point" in the division of decimals.

Checking in Division. Multiply the divisor by the answer (quotient) to get the number which was divided originally. If there was a remainder add as indicated below.

```
        Problem                  Check
                 82                  197
          197)16158                   82
              1576                   ---
              ----                   394
               398                 1576
               394                 -----
               ---                 16154
Remainder        4     Remainder       4
                                   -----
                                   16158
```

ILLUSTRATIVE PROBLEMS

Following are a few illustrative problems requiring the application of the so-called fundamental operations, — addition, subtraction, multiplication and division. Observe the proper method of writing the answer.

When we add 2, 5 and 10 we write simply the number 17 as the answer. If, however, we add 2 bolts, 5 bolts and 10 bolts the answer is 17 bolts. When you add real things the answer should contain the name of the things added. The same idea holds true in subtraction.

In multiplication the method of securing the answer is as follows: When we multiply 5 motors by 2, we are dealing with 2 groups of motors, each containing 5 motors, or a total of 10 motors. Remember that multiplication is merely a short cut in addition. The same answer would be obtained if 5 motors and 5 motors were added.

The same is true in division. When we divide a quantity we find into how many parts or groups the total quantity is divided. If we divide 1600 feet of wire by 4, the answer should be written *400 feet of wire* in each lot. If the problem were to find how many pieces of wire, each 4 feet long, could be cut from 1600 feet of wire, the answer would be *400 pieces.*

Example. A keg contains 50 pounds of nuts and washers. Of this amount there are 20 pounds of washers and 30 pounds of nuts. If there are 30 washers in a pound and 18 nuts in a pound, how many washers and nuts are there in the keg?

Solution. As there are 20 pounds of washers and there are 30 washers in a pound, the total number of washers will be 20 times 30 washers or 600 washers.

As there are 30 pounds of nuts and there are 18 nuts in a pound, the total number of nuts will be 30 times 18 nuts or 540 nuts.

Ans. 600 washers; 540 nuts.

Example. For a certain construction job five pieces of floor joist are needed. The required lengths of the pieces are as follows: 11 feet, 8 feet, 6 feet, 4 feet, and 3 feet. What is the total length to be purchased, allowing 1 foot for waste?

Solution. 11 feet plus 8 feet plus 6 feet plus 4 feet plus 3 feet equals 32 feet. This is the actual amount required. Since 1 foot more must be purchased to allow for waste, 33 feet must be purchased.

Ans. 33 feet.

Example. In a motion picture camera 10 turns of the crank move 5 feet of the film. How many feet of film pass a given point in 3 turns of the crank?

Solution. If 10 turns of the crank are required to move 5 feet of film past the point, in order to find how much film is moved by 1 turn of the crank, we divide 5 by 10, and get $\frac{5}{10}$ or $\frac{1}{2}$ of a foot. In 3 turns of the crank, 3 times $\frac{1}{2}$ of a foot ($\frac{3}{2}$), or $1\frac{1}{2}$ feet, will be drawn past the point.

Example. Is it cheaper to hire a boy at $17\frac{1}{2}$ cents per hour for 15 hours for a certain piece of work or a man at 55 cents per hour who can do the work in $4\frac{1}{2}$ hours?

Solution. 15 times $17\frac{1}{2}$ cents will give the number of cents necessary to pay the boy. $15 \times 17 = 255$; $15 \times \frac{1}{2} = \frac{15}{2} = 7\frac{1}{2}$; 255 cents $+ 7\frac{1}{2}$ cents $= 262\frac{1}{2}$ cents. That is, \$2.62½ will be required to pay the boy.

The man will require $4\frac{1}{2} \times 55$ cents for his pay. $4 \times 55 = 220$; $\frac{1}{2} \times 55 = 27\frac{1}{2}$; $220 + 27\frac{1}{2} = 247\frac{1}{2}$. That is, \$2.47½ will be needed to pay the man for his labor. It will be cheaper to hire the man.

Example. An electric motor weighs 1080 pounds. The foundation for it is to be strong enough to support $1\frac{1}{2}$ times the weight of the motor. For how much weight must the foundation be designed?

Solution. If the motor weighs 1080 pounds it is evident in this case that the foundation must be strong enough to support a weight one and one-half times 1080. This is equivalent to adding $\frac{1}{2}$ of 1080 to 1080. $\frac{1}{2}$ of 1080 pounds = 540 pounds.

1080 pounds + 540 pounds = 1620 pounds. The foundation must be strong enough to support this weight.

Units of Measurement are used so frequently that it seems best to include them here. Those most commonly used should be memorized.

Linear or Long Measure

12 inches (in.)	make 1 foot (ft.)
3 feet	make 1 yard (yd.)
$16\frac{1}{2}$ feet	make 1 rod (rd.)
5280 feet	make 1 mile (mi.)

Square Measure

144 square inches (sq. in.)	make 1 square foot (sq. ft.)
9 square feet	make 1 square yard (sq. yd.)
43,560 square feet	make 1 acre (A.)
640 acres	make 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.)	make 1 cubic foot (cu. ft.)
27 cubic feet	make 1 cubic yard (cu. yd.)
128 cubic feet	make 1 cord of wood (a pile $4 \times 4 \times 8$ ft.)
231 cubic inches	make 1 gallon
7.48 U.S. gallons (gal.)	make 1 cubic foot
$24\frac{3}{4}$ cubic feet	make 1 perch of masonry ($16\frac{1}{2} \times 1\frac{1}{2} \times 1$ ft.)

Time Measure

60 seconds (sec.)	make 1 minute (min.)
60 minutes	make 1 hour (hr.)
24 hours	make 1 day (da.)
7 days	make 1 week (wk.)
$365\frac{1}{4}$ days	make 1 year (yr.)

Avoirdupois Weight

Used to weigh all metals except gold and silver.

16 ounces (oz.)	make 1 pound (lb.)
2000 pounds	make 1 short ton (T.)
2240 pounds	make 1 long ton

United States Dry Measure

2 pints (pt.)	make 1 quart (qt.)
8 quarts	make 1 peck (pk.)
4 pecks	make 1 bushel (bu.)

The Winchester bushel, which is a cylinder $18\frac{1}{2}$ inches in diameter and 8 inches deep, is the standard U. S. bushel. It contains 2150.42 cubic inches. A pint is equal to 33.60 cubic inches.

Circular Measure

60 seconds	make 1 minute
60 minutes	make 1 degree (°)
90 degrees	make 1 quadrant (a right angle)
360 degrees	make 1 circumference

Problems — Group I

1. A railroad track is 26 miles long. How many rails each 30 feet long will be required to lay the track?

2. If you allow $1\frac{1}{16}$ inches of wire for each nail, how many feet of wire will be required to make 500 nails?

3. Find the sums of the following columns of figures:

4568	15431	7386	49850	6452	62165
7391	29685	45371	17370	63834	16732
7854	73648	13764	68429	76343	85696
53469	34519	9887	23156	80931	71883
13470	78243	64348	21017	79883	50149
58143	7843	14627	67154	83578	31572
			64353	35647	76844

Add each of the above groups in the usual way, up and down; then add each line of figures horizontally, placing the sums at the right as a seventh group. The sum of the figures in this last group should equal the sum of all the answers to the six groups first added.

4. Two pieces of wood each 2 inches thick (Fig. 2) are joined together by two screws each $3\frac{5}{8}$ inches long. How far will the

ends of the screws be from the side opposite that from which they enter?

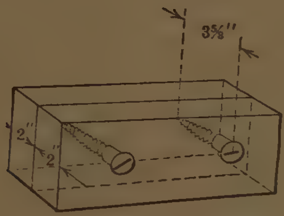


FIG. 2. Wooden Blocks.

5. How much molding is required for a picture frame whose outside dimensions are 13 inches by 17½ inches?

6 If the cost of construction of 375 miles of state road was $4,195,400, what was the average cost per mile?

7. A certain wall exerts a pressure on the soil of 225 pounds per square foot. What pressure would this be in ounces per square inch?

8. A room in a factory is lighted by means of 68 gas burners each of which consumes 22½ cubic feet of gas per hour. If gas costs 85 cents per thousand cubic feet what will it cost to light the room for 2½ hours?

9. If 26 feet of telephone wire weigh one pound, what will be the weight of a mile of such wire?

10. Find the number of 26-inch lengths that could be obtained from sawing up a board 11 feet long, making no allowance for waste.

11. It is estimated that in order to do satisfactory work the teeth of a saw must travel about 9000 feet per minute. How many miles per minute would this be?

12. A bin filled with lime contains 245 bushels. This is to be put into sacks for transportation. If each sack contains 2 bushels and 1 peck, how many sacks will be required?

13. How long would it take a man riding in an automobile to cover a distance of 3000 miles at an average rate of 23 miles an hour. Express answer in days, hours, minutes, and seconds.

14. If a piece of electric cable 10 feet $3\frac{1}{4}$ inches long is cut into three equal pieces and the width of saw cut is $\frac{1}{8}$ of an inch, how long will each piece be? Note that allowance must be made for two saw cuts and that 2 times $\frac{1}{8}$ of an inch equals $\frac{1}{4}$ of an inch.

15. The cost of laying cement is usually figured by the square yard. What will it cost per square yard to make a cement walk 300 feet long and 5 feet wide if the cost for labor is $162.30, for cement $92.00 and for gravel $42.50?

16. A carpenter wishes to cut four shelves each 2 feet 9 inches long. There are three boards available for this. One of these is 10 feet long, another 12 feet long and another 14 feet long. Which board will cut with the least waste and how much waste will there be?

17. There are 2 miles and 1254 feet of insulated copper wire in a spool. If this wire is cut into pieces each of which is 2 feet 8 inches long, how many pieces are obtained?

18. Choose any number; multiply it by 6; add 12; divide by 6; subtract the original number. What is the result? To prove that your result is correct, work the problem several times using a different number each time.

19. Choose any number; multiply it by 10; subtract 5; divide by 5; add 1. What is the relation of the answer to the original number. Check as in problem 18.

20. How many posts will be required to fence in a field 675 feet long and 324 feet wide if the posts are placed 9 feet apart? Posts must be at each corner.

Problems — Group II

1. The shaft of a 20-horsepower motor must be of sufficient length to have 10 inches inclosed in bearings, 15 inches covered by the armature, 6 inches covered by the pulley hub, and 3 inches total between these parts. How long is the shaft?

2. A steam-electric power plant contains a generator weighing 231,517 pounds, an engine weighing 431,286 pounds, boilers weighing 211,138 pounds, and pumps, pipes, and other articles weighing 106,459 pounds. What is the total weight of the apparatus?

3. A telephone company wished to know exactly how many poles there were in a certain district. The district was divided into five sections, and a man was assigned to each section to count the poles. These men reported as follows: 10,326, 5487, 16,325, 4812 and 7608. How many poles were in the district?

4. An electrician kept a record of the amount of wire required for a house and found the requirements of the various rooms to be as follows: parlor 275 feet; dining room, 316 feet; kitchen, 103 feet; library, 167 feet; basement, 308 feet; chambers on second floor, 106, 151, 123, 75 and 186 feet; chambers on third floor, 86, 173, 119 and 98 feet; connecting wires and entrance wires, 324 feet. How many feet of wire were required for the house?

5. A reel contained 1037 feet of wire. 832 feet were used for wiring a house and the remainder for wiring a barn. How many feet of wire were used for the barn?

6. A generator complete weighs 3000 pounds. Without the base the weight is 2474 pounds. How much does the base weigh?

7. A factory is illuminated by 684 lamps. Of this number 249 are carbon lamps and the remainder are tungsten lamps. How many tungsten lamps are there?

8. An electrician with a helper completed a wiring contract in 246 hours. Without the helper he estimated that it would take 294 hours. How many hours did he save by having a helper?

9. On a certain job 80 men worked 60 hours each, 62 men worked 55 hours each, 18 men worked 46 hours each and one man worked 49 hours. What was the total number of hours worked by all of the men?

10. Each room in a hotel requires 6 tungsten lamps. There are 7 floors and 22 rooms on each floor. How many lamps are required for the hotel?

11. A laundry is equipped with 3 electric fans taking 214 watts each, 2 flat irons taking 500 watts each and 6 tungsten lamps taking 40 watts each. How many watts are taken by the laundry?

12. In wiring a building it was found that 4 rooms required 3 switches each, 5 rooms required 2 switches each, 6 rooms required 1 switch each and 1 room required 5 switches. How many switches were required for this building?

13. In three years a telephone factory shipped 2,477,475 telephones. 819 cars were required. How many telephones were shipped in each car?

14. A carload of electric fans weighed 49,324 pounds. These fans were packed in boxes with 19 fans to the box. The car held 118 boxes. What was the weight of each fan?

15. A street railway company contracted for 4,756,914 pounds of rails to be delivered in equal amounts. The deliveries had to be received three times per month for a period of nine months. How many pounds were delivered in each lot?

CHAPTER II

COMMON FRACTIONS AND THEIR APPLICATION

Explanation of Fractions. The study of fractions is simple when one understands their meaning. When a pile of coal is divided into four parts, as must be done in coal testing (Fig. 3), each of the four parts is a *fraction* of the whole pile, and each of the parts is called a quarter. In this case one quarter (also written $\frac{1}{4}$ and read one fourth) is taken away for testing and three quarters (also written $\frac{3}{4}$ and read three fourths) are thrown back into the coal bin. Any number of these parts may be written as a fraction. Thus 2 parts equal $\frac{2}{4}$ of the whole, 3 parts equal $\frac{3}{4}$ of the whole. $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, etc., are all called fractions. Similarly, $\frac{5}{8}$ is a fraction, and means five of the eight equal parts into which something has been divided; also $\frac{7}{16}$ means seven of the sixteen equal parts.

FIG. 3. Fractions of a Coal Pile.

The meaning of a fraction may be well illustrated with an ordinary ruler. Fig. 4 shows a small ruler, three inches long. As shown, each inch is divided into sixteen equal parts and each one of these parts is, therefore, called one

sixteenth ($\frac{1}{16}$) of an inch. Remember that sixteen sixteenths, which is written $\frac{16}{16}$, means sixteen divided by sixteen, which equals 1. Any number divided by itself equals 1.

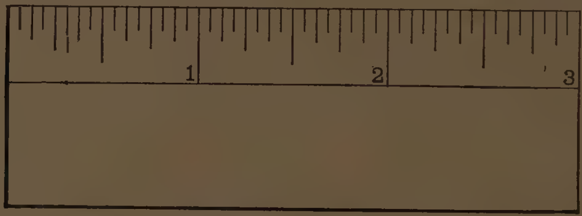


FIG. 4. Ruler showing Fractions of an Inch.

Numerator and Denominator. A common fraction has two parts, the numerator and the denominator. These parts are separated by a dividing line as $\frac{2}{3}$, sometimes written 2/3. The denominator shows into how many parts the quantity is divided; the numerator indicates how many parts are taken. By custom, the numerator is always placed above the line, the denominator below the line.

Take the fraction $\frac{3}{8}$ for further explanation and the block of wood (Fig. 5) as an illustration. This block as shown has

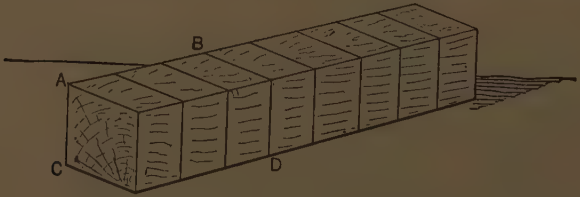


FIG. 5. Fractions of a Wooden Block.

been sawed into eight equal parts. Three of these parts are marked by the letters *ABCD* which are $\frac{3}{8}$ of the whole block. The denominator 8 shows the number of parts into which

the whole block has been divided and the numerator 3 tells how many of these parts are included in the marked or lettered portion.

It is sometimes convenient to use the names *proper fraction* and *improper fraction.* In a proper fraction the numerator is less than the denominator; in an improper fraction the numerator is equal to, or greater than, the denominator.

Addition of Fractions. It has already been suggested that we cannot add unlike quantities, as, for instance, dollars and doughnuts. Similarly, we cannot add unlike fractions, that is, fractions which do not have the same denominator. For example, the fractions $\frac{1}{4}$ and $\frac{1}{8}$ cannot be added until they have been changed or reduced so as to have the same denominator. The method of adding fractions may be illustrated concretely by calculating what part or fraction of a gallon of paint we get by adding together as in Fig. 6 one

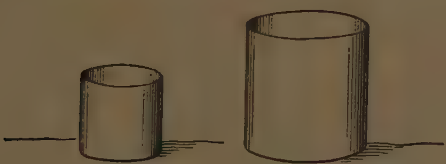


FIG. 6. Fractions of a Gallon.

quart ($\frac{1}{4}$ gallon) and one pint ($\frac{1}{8}$ gallon). Obviously, we can make the denominators of these two fractions the same by remembering that $\frac{1}{4}$ is the same as $\frac{2}{8}$ and that, therefore, $\frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8}$. Now these two fractions having the same denominator are "of the same kind," and we can add together any number of things "of the same kind," so that two eighths added to one eighth makes *three* eighths, or $\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$. Answer, $\frac{3}{8}$ of a gallon. From this it will be seen that in order to add fractions we must first change

them so that all have the *same denominator* and then find the *sum* of their *numerators.*

In work with fractions it is not always so easy as in this case to see the change which must be made to get the same denominator. The following principle will, however, be of great assistance and should be remembered.

Multiplying or dividing both the numerator and the denominator of a fraction by the same number does not change the value of the fraction.

If this principle is applied to the above example, we can see that by multiplying both the numerator and denominator of $\frac{1}{4}$ by 2 we get the following:

$$\frac{1 \times 2 = 2}{4 \times 2 = 8}.$$

In many cases the same, or common, denominator cannot be found, as in the last case, *by observation.* For instance, if we wish to add $\frac{5}{16}$, $\frac{5}{9}$, $\frac{3}{12}$, we must first find the smallest number that will divide without remainder at least two of the

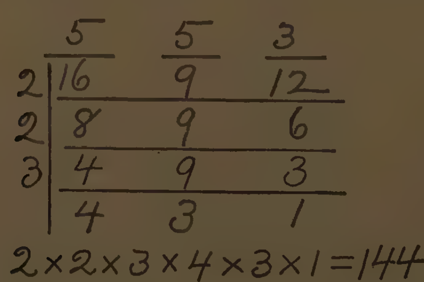


FIG. 7. Method of Finding Smallest (Lowest) Common Denominator.

denominators of the fractions to be added. Then divide by this number all the denominators that are exactly divisible by it. Do this division somewhat as illustrated in Fig. 7.

Write below the denominators the quotients from the exactly divisible numbers and also the numbers themselves for those which are not exactly divisible by the divisor used. Repeat this division as shown in the figure until no number can be found which will exactly divide any two of the denominators. Then the product of the several divisors and the numbers remaining in the various columns after the last division, multiplied together, will give $2 \times 2 \times 3 \times 4 \times 3 \times 1 = 144$, which is the smallest (lowest) common denominator. All the fractions must, therefore, be changed to have 144 for the denominator in order that they may be added.

The next step is to find out how many 144ths are equal to $\frac{5}{16}$. Since 16 is contained in 144 nine times, $\frac{1}{16}$ equals $\frac{9}{144}$, and $\frac{5}{16}$ equals $\frac{5 \times 9}{144} = \frac{45}{144}$. Similarly, $\frac{1}{9} = \frac{16}{144}$ and $\frac{5}{9} = \frac{80}{144}$. In the same way, we find that $\frac{3}{12}$ equals $\frac{36}{144}$. The sum of $\frac{5}{16} + \frac{5}{9} + \frac{3}{12} = \frac{45}{144} + \frac{80}{144} + \frac{36}{144} = \frac{161}{144}$. Usually when the numerator is larger than the denominator we change the fraction into a whole number and a fraction, and in this case we shall have $1\frac{17}{144}$.

Rules for finding the Divisors of Denominators. It is often convenient to tell by observation, without performing the division, whether or not the number is divisible by another. The following rules are easily applied and will often save time in finding common denominators.

(1) A number is divisible by 2 if its right-hand figure is 0 or is divisible by 2.

(2) A number is divisible by 3 if the sum of its figures is divisible by 3. Thus 73,245 is divisible by 3 since $7 + 3 + 2 + 4 + 5 = 21$ is divisible by 3.

(3) A number is divisible by 4 if the number represented by its last two figures on the right is divisible by

4, or if it ends in two zeros. Thus, 87,656 is divisible by 4 since 56 is divisible by 4.

(4) A number is divisible by 5 if the last figure on the right is 0 or 5.

(5) When the sum of the figures in an *even* number is divisible by 3 the number is divisible by 6.

(6) A number is divisible by 8 if the number represented by the last three figures on the right is divisible by 8. Thus 987,672 is divisible by 8 since 672 is divisible by 8.

(7) A number is divisible by 9 if the sum of its figures is divisible by 9.

The above rules will be found useful in all kinds of calculations involving division.

Changing Numbers to Fractions. In the simplest case, a *whole number* is changed to a fraction by multiplying the number by the denominator which the fraction is to have. This multiplication gives the numerator of the required fraction. For example, if the number 5 is to be changed to what it is equal to in "eighths," we can get the numerator of the fraction by multiplying 5 by 8. We know that 1 equals eight eighths, therefore, the number $5 = 5 \times 8$ eighths, or 40 eighths, which may also be written $\frac{40}{8}$. Similarly we can change $6\frac{3}{8}$ to eighths as follows: The number 6 equals six times 8 eighths $= \frac{48}{8}$, and $\frac{48}{8} + \frac{3}{8} = \frac{51}{8}$.

Changing Fractions to Numbers. Sometimes fractions are too large for the use we want to make of them. In that case we change them to equal values composed of a whole* number and a fraction. Thus if we have a fraction $\frac{64}{4}$ we can reduce it to a whole number by dividing the numerator

* The term *whole number* as used in this connection means a number without a fraction.

and denominator by the same number as the denominator, thus,

$$\frac{64 \div 4 = 16}{4 \div 4 = 1} \text{ or } 16. \quad \text{Similarly } \frac{35 \div 6 = 5\frac{5}{6}}{6 \div 6 = 1} \text{ or } 5\tfrac{5}{6}.$$

By a short-cut method which will give this same result, we simply divide the numerator by the denominator, as $\frac{64}{4} = 64 \div 4 = 16$; or $\frac{35}{6} = 35 \div 6 = 5\frac{5}{6}$.

Reducing Fractions. Usually it is convenient to reduce a fraction having a large numerator or a large denominator, or both, to a simple fraction composed of smaller figures. This reducing of fractions consists of dividing both the numerator and the denominator by a number which exactly divides both of them. Thus if the fraction $\frac{960}{720}$ is to be reduced we must first find a divisor of both the numerator and the denominator. In the first place, both are divisible by 10, thus

$$\frac{960 \div 10 = 96}{720 \div 10 = 72}.$$

Again, both the numerator and the denominator of this simplified fraction are divisible by 24, and we have further:

$$\frac{96 \div 24 = 4}{72 \div 24 = 3}.$$

With practice we should have seen that both terms were divisible by 240.

Mixed number as used in mathematics means a whole number combined with a fraction, like $3\frac{1}{3}$.

Addition of Numbers and Fractions (*Mixed Numbers*). There are two ways of adding combined numbers and fractions. If $7\frac{5}{16}$ is to be added to $2\frac{5}{8}$, we might do it by

(1) Adding first the whole numbers and then the fractions, or

(2) Reducing the whole numbers with their fractions to single fractions.

By the first of these two ways we should have by the addition of the whole numbers in the example above, $7 + 2 = 9$ and by the addition of the fractions $\frac{5}{16} + \frac{5}{8} = \frac{5}{16} + \frac{10}{16} = \frac{15}{16}$. Then, when the sum of the whole numbers and the sum of the fractions are combined, we shall have $9\frac{15}{16}$.

The second way is to reduce the first number and its fraction to sixteenths, thus $7\frac{5}{16}$ equals $\frac{117}{16}$. Similarly $2\frac{5}{8}$ equals $\frac{21}{8}$, or $\frac{42}{16}$. Adding $\frac{117}{16}$ and $\frac{42}{16}$ we have $\frac{159}{16}$ which equals $9\frac{15}{16}$. The same result is obtained by either method. Usually, however, the first method is preferred in practical work because it is simpler and more rapid, especially when the fractions involved are familiar, as $4\frac{1}{8}$, $2\frac{3}{4}$, $6\frac{3}{5}$, etc.

Subtraction of Numbers and Fractions (*Mixed Numbers*). The same two methods explained for the addition of numbers combined with fractions can be used for subtraction. Take the same two numbers and subtract $2\frac{5}{8}$ from $7\frac{5}{16}$. After writing both numbers as fractions with the same denominator, according to the first of the ways explained for addition, we should have $7\frac{5}{16}$ and $2\frac{10}{16}$. In this case, however, we cannot subtract the fractions as they stand because $\frac{5}{16}$ is smaller than $\frac{10}{16}$. We can, however, write $7\frac{5}{16}$ as $6\frac{5}{16} + \frac{16}{16}$ or $6\frac{21}{16}$. In this form we can subtract $\frac{10}{16}$ from $\frac{21}{16}$ leaving $\frac{11}{16}$, and we can also subtract 2 from 6 leaving 4, so that the final result is, therefore, $4\frac{11}{16}$.

The second method may also be used in the subtraction of fractions. For example, $7\frac{5}{16} = \frac{117}{16}$ and $2\frac{5}{8} = \frac{42}{16}$. Subtracting the smaller from the larger the difference is $\frac{75}{16}$, which equals $4\frac{11}{16}$.

Multiplication of Fractions. When multiplying fractions it is necessary to keep in mind two principles, as follows:

1. A fraction may be multiplied by a whole number (*a*) by multiplying the numerator by that number or (*b*) by dividing the denominator by that number. Following are examples of the use of these principles:

$$(a)\ \frac{3}{20} \times 7 = \frac{3 \times 7}{20} = \frac{21}{20} = 1\frac{1}{20}.$$

Also $$\frac{5}{12} \times 2 = \frac{5 \times 2}{12} = \frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}.$$

$$(b)\ \frac{5}{12} \times 2 = \frac{5}{12 \div 2} = \frac{5}{6}.$$

2. When we wish to multiply one fraction by another fraction, we multiply together the two numerators and the two denominators. The multiplication of the numerators will form the numerator of the answer, and the multiplication of the denominators will form the denominator of the answer. For example

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12}.$$

For a practical application note the following: If a dam was $\frac{4}{5}$ full of water and $\frac{1}{2}$ of this water was used by a power plant, we know that $\frac{1}{2}$ the *four* fifths ($\frac{4}{5}$) or *two* fifths ($\frac{2}{5}$) of the water was taken out. This result could have been obtained by the second method by writing for the numerator of the result the multiplied numerators of the two fractions, and by writing for the denominator of the result the multiplied denominators of the two fractions, thus,

$$\frac{4 \times 1}{5 \times 2} = \frac{4}{10} = \frac{2}{5}.$$

3. A mixed number (see page 21) may be multiplied by a fraction by reducing the mixed number to a single fraction and then multiplying as follows:

$$5\tfrac{7}{12} \times \tfrac{2}{3} = \tfrac{67}{12} \times \tfrac{2}{3} = \tfrac{134}{36} = 3\tfrac{26}{36} = 3\tfrac{13}{18}.$$

Similarly, $2\tfrac{2}{3} \times 3\tfrac{1}{2} = \tfrac{8}{3} \times \tfrac{7}{2} = \tfrac{56}{6} = 9\tfrac{2}{6} = 9\tfrac{1}{3}.$

Division of Fractions. 1. A fraction may be divided by a whole number either by dividing the numerator by that number, or by multiplying the denominator by that number. The following examples will illustrate this:

Divide $\tfrac{3}{20}$ by 3:

$$\frac{3 \div 3}{20} = \frac{1}{20}; \quad \text{or} \quad \frac{3}{20 \times 3} = \frac{3}{60} = \frac{1}{20}.$$

2. A fraction may be divided by another fraction most simply by inverting * the divisor (the dividing fraction) then multiplying together the numerators and denominators as in the multiplication of fractions.

For example $\tfrac{3}{4}$ divided by $\tfrac{4}{5}$ is solved most simply by multiplying the fraction $\tfrac{3}{4}$ by the fraction $\tfrac{4}{5}$ inverted, thus,

$$\tfrac{3}{4} \div \tfrac{4}{5} = \tfrac{3}{4} \times \tfrac{5}{4} = \tfrac{15}{16}.$$

3. A mixed number can be divided by a fraction by changing the mixed number to a fraction and following the directions for division of fractions, thus:

$$5\tfrac{7}{12} \div \tfrac{2}{3} = \tfrac{67}{12} \times \tfrac{3}{2} = \tfrac{201}{24} = 8\tfrac{9}{24} = 8\tfrac{3}{8}.$$

Similarly,

$$3\tfrac{7}{12} \div 3\tfrac{2}{3} = \tfrac{43}{12} \times \tfrac{3}{11} = \tfrac{129}{132} = \text{(dividing by 3) } \tfrac{43}{44}.$$

Cancellation. In many cases the multiplication of fractions may be simplified by dividing any numerator by any denominator of a fraction to be multiplied, or dividing any

* To invert a fraction is to write it upside down, making the numerator and the denominator change places.

denominator by any numerator. If we wish, the same principle may be extended to include dividing any numerator and any denominator by the same number. This method of simplifying the multiplication of fractions is called cancellation, for example,

$$2\tfrac{2}{3} \times \tfrac{3}{4} = \tfrac{8}{3} \times \tfrac{3}{4} = \text{which may be written } \frac{\overset{2}{\cancel{8}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{4}}} = 2.$$

Take as another example, $5\tfrac{9}{12} \times 3\tfrac{1}{3} \times 2\tfrac{2}{5} = \tfrac{69}{12} \times \tfrac{10}{3} \times \tfrac{12}{5} =$

Dividing first by 4, then by 5, then by 3,

$$\frac{\overset{23}{\cancel{69}} \times \overset{2}{\cancel{10}} \times \overset{\overset{1}{\cancel{3}}}{\cancel{12}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{12}} \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}}} = 23 \times 2 = 46.$$

You could, of course, have divided by 12, by 3, by 5 with the same result.

Remember that when you divide one or more numerators by a given number you must divide exactly as many denominators by the same number. Do not forget after dividing to put down the result even if it is only 1.

The rules given in Chapter I for checking your work may be applied to fractions in the same way as to whole numbers, that is, check addition by first adding upward and then downward, and if you have numbers of several figures, by adding the columns separately and then adding these totals as for example:

Problem — add

$$\begin{array}{r} 14\tfrac{7}{8} \\ 2\tfrac{1}{2} \\ 7\tfrac{1}{3} \\ \underline{943\tfrac{3}{4}} \end{array}$$

The whole numbers total 966. Add the fractions, after first changing them to fractions having a common denominator, 24.

Making this change we have

$$\frac{7}{8} \text{ equals } \frac{21}{24}$$
$$\frac{1}{2} \text{ equals } \frac{12}{24}$$
$$\frac{1}{3} \text{ equals } \frac{8}{24}$$
$$\frac{3}{4} \text{ equals } \frac{18}{24}$$

Then $\frac{21}{24} + \frac{12}{24} + \frac{8}{24} + \frac{18}{24} = \frac{59}{24}$.

Since $\frac{59}{24}$ is the same as $2\frac{11}{24}$ we have $966 + 2\frac{11}{24} = 968\frac{11}{24}$.

The rules in Chapter I for checking subtraction, multiplication and division of whole numbers apply equally well to fractions.

You should always check your work. Practice on the problems which follow.

Illustrative Problems

Example. If $12\frac{1}{2}$ bushels of charcoal cost \$3.25 ($3\frac{1}{4}$ dollars), find the price per bushel. As we are to find the price of 1 bushel, the number $12\frac{1}{2}$, denoting bushels, is the *divisor*.

Solution. —

$$3\frac{1}{4} \div 12\frac{1}{2} = \frac{13}{4} \div \frac{25}{2} = \frac{13 \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times 25} = \frac{13}{50} \text{ of a dollar per bushel.}$$

Example. If 12 pounds of nails can be bought for 96 cents, how much can be bought for 4 cents?

Solution. If 96 cents will buy 12 pounds, then 4 cents will buy $\frac{4}{96}$ as much, or $\frac{4}{96}$ of 12 pounds, or $\frac{48}{96}$ pounds, or $\frac{1}{2}$ pound for 4 cents.

Example. Which is the greater, $\frac{3}{4}$ or $\frac{5}{6}$?

Solution. As the lowest common denominator is 12, $\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$. As $\frac{10}{12}$, the equivalent of $\frac{5}{6}$, is greater than $\frac{9}{12}$, the equivalent of $\frac{3}{4}$, $\frac{5}{6}$ is greater than $\frac{3}{4}$.

Example. If an oil engine requires $\frac{7}{8}$ of a pint of oil per horse power per hour, what will be the cost of running a 5-horse-power engine for 8 hours with oil at \$18.24 per barrel? (Take a barrel as $31\frac{1}{2}$ gallons.)

Solution. There are 8 pints in a gallon. This means of course that there will be $31\frac{1}{2}$ times 8 pints in a barrel or 252 pints. In \$21.42 there are 2142 cents. $2142 \div 252$ will give cost per pint or $8\frac{1}{2}$ cents. If one pint costs $8\frac{1}{2}$ cents, $\frac{7}{8}$ pint will cost $\frac{7}{8}$ of $8\frac{1}{2}$ cents.

$$\tfrac{7}{8} \text{ of } 8\tfrac{1}{2} = \tfrac{7}{8} \times \tfrac{17}{2} = \tfrac{119}{16} = 7\tfrac{7}{16} \text{ cents.}$$

The cost per horse power per hour is therefore $7\frac{7}{16}$ cents.

Since the engine is 5 horse power, the cost of running the engine per hour will be 5 times $7\frac{7}{16}$ cents or $37\frac{3}{16}$ cents.

If the engine runs for 8 hours the entire cost of oil required will be 8 times $37\frac{3}{16}$ cents or \$2.97$\frac{1}{2}$.

Example. An electrician is asked to divide the amount of a bill due him for light installation in such a way that $\frac{3}{7}$ of it is to be paid by one man and $\frac{4}{7}$ by another. The amount of the bill is \$70.63.

Solution. Since there are seven sevenths ($\frac{7}{7}$) in a whole one, the first thing to do here is to find one seventh ($\frac{1}{7}$) of \$70.63.

$$\text{Thus } \$70.63 \times \frac{1}{7} = \$\frac{70.63}{7} = \$10.09.$$

Then $\frac{3}{7}$ will be $3 \times \$10.09$ or \$30.27

and $\frac{4}{7}$ will be $4 \times \$10.09$ or \$40.36.

Check: $\$30.27 + \$40.36 = \$70.63$.

Problems — Group I

1. How many $\frac{3}{4}$-inch strips can be cut from a piece of sheet zinc 9 inches wide?

2. How many $1\frac{1}{2}$ pint measures (Fig. 8) of oil can be poured from a 5-gallon can?

3. Four men purchased an electrical supply business. The first paid for $\frac{3}{10}$ of it, the second for $\frac{375}{1000}$ of it and the third for $\frac{1}{5}$ of it and the fourth man paid for the remainder, which was \$3000. How much did the business cost each of the first three men? What was the total cost?

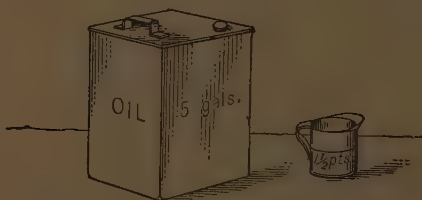


FIG. 8. Oil Measure and Oil Can.

4. How many miles will an electric freight locomotive (Fig. 9) travel in $5\frac{5}{6}$ hours, if it travels $20\frac{3}{4}$ miles per hour?

5. A certain firm found after investigation that employees with a common school education earned on an average \$1000 per year, those with a high school education $\frac{4}{5}$ more, and those with technical education $\frac{2}{5}$ more than the high school graduates. What is the average yearly salary of the high school graduates? of the technical men?

6. An airplane covered 550 miles in $4\frac{1}{6}$ hours. What was its rate per hour?

7. Sound travels 1120 feet per second. Thunder is heard 11 seconds after a lightning flash is seen. How many miles away is the thunder clap?

8. If a spark plug for an automobile engine retails for \$1.50 what part of the cost of it to the purchaser may be charged to the items *a*, *b*, *c* and *d* below?

a. $\frac{2}{5}$ of the sale price for material,
b. $\frac{4}{15}$ of the sale price for labor,
c. $\frac{1}{6}$ of the sale price for wholesaler's profit,
d. What is the retailer's profit?

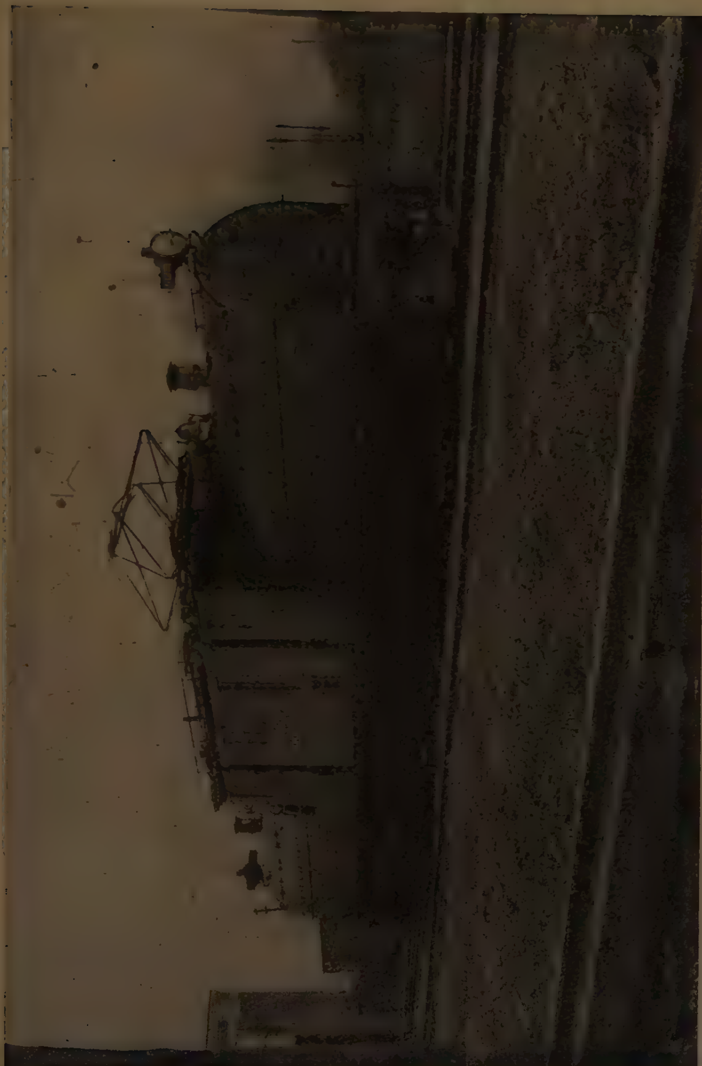


Fig. 9. Electric Locomotive on Chicago, Milwaukee, and St. Paul Railroad.

9. If the dimensions A and B of a clutch yoke, Fig. 11, are the same size what is each equal to? (This is a problem in subtraction.)

Fig. 10. Spark Plug.

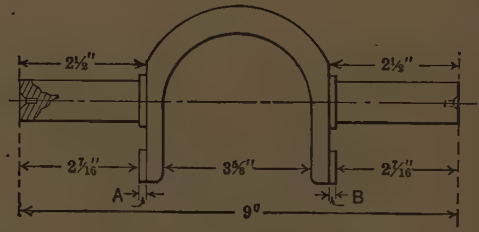


Fig. 11. Clutch Yoke.

10. If the outside diameter of a pipe (Fig. 12) is $2\frac{1}{4}$ inches and the inside diameter $1\frac{7}{8}$ inches, how thick is the pipe?

11. If it takes $17\frac{1}{2}$ minutes to pump an automobile tire to 70 pounds pressure with the pump in Fig. 13 and it requires 375 strokes to do it, how many seconds are required for each stroke?

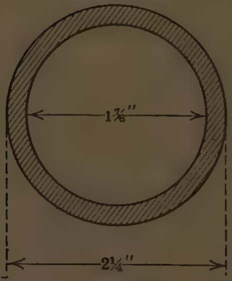


Fig. 12. Cross-section of Pipe.

Fig. 13. Automobile Air Pump.

12. If the circumference, or distance around a circle, is $\frac{22}{7}$ times the diameter of the circle, what fractional part of the circumference is the diameter?

13. What is the distance *P* between the threads of the cap screw in Fig. 14? This distance is called the *pitch*. Count the number of *spaces* between the threads for the dimension given.

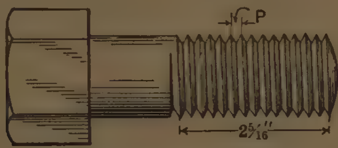


FIG. 14. Cap Screw showing Pitch of Threads.

14. The beam in Fig. 15 has a uniform load of 1050 pounds between the supports shown, what is the average load per foot of length? The distance between the supports is 18 feet 6 inches.

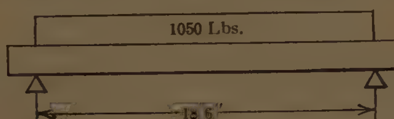


FIG. 15. Beam with Uniform Load.

PROBLEMS — GROUP II

1. An electrician completes $\frac{2}{6}$ of a job on one day and $\frac{1}{4}$ of it on the following day. What part of the whole job is completed? What part remains to be done?

2. The total length of the shaft of an electric fan is $11\frac{5}{8}$ inches. $6\frac{1}{8}$ inches of the shaft are allowed for the armature, $2\frac{1}{4}$ inches for the bearings, $1\frac{1}{16}$ inches for the fan hub, and the remainder is clear. How many inches are clear?

3. A barrel of porcelain knobs is $\frac{5}{8}$ full. Another barrel of the same size is $\frac{7}{24}$ full. If all the knobs were in one barrel, how full would it be?

4. A workman can assemble $\frac{10}{32}$ of a motor in one day. How many motors can he assemble in $3\frac{2}{5}$ days?

5. An electrician finds that because of limited space he can use only $\frac{3}{4}$ of a bolt $\frac{11}{24}$ inch long. How long is the part he uses?

6. The cost of repairs on a transformer is \$21, which is $\frac{7}{24}$ of the entire cost of repairs. What is the entire cost of repairs?

7. If a layer of nickel plate $\frac{3}{64}$ inch thick can be deposited by an electric current in one hour, in how many hours will a deposit $\frac{7}{32}$ inch thick be made?

8. If an electrician earns \$$2\frac{1}{4}$ per day, how long must he work to earn \$$5\frac{1}{4}$?

9. A piece of bare copper wire weighs $\frac{9}{16}$ ounce per inch. How many inches will weigh $15\frac{21}{256}$ ounces?

10. How much does a can of oil holding $2\frac{49}{68}$ gallons weigh, if $\frac{4}{17}$ gallon weighs one pound?

11. It costs $5\frac{1}{16}$ cents to run a motor for 1 hour. What does it cost to run it for $19\frac{4}{9}$ hours?

12. A generator weighs $125\frac{7}{9}$ pounds. Its concrete foundation weighs $\frac{9}{16}$ as much as the generator. What is the weight of the foundation?

13. A generator requires $\frac{3}{64}$ pint of oil per hour. If it runs 50 hours per week, how much oil will it use in 52 weeks or one year?

14. The cost of repairs on a motor is \$12 which is found to be $\frac{3}{8}$ of the total cost of repairs for the month. What is the cost of repairs for the month?

15. Three lengths of wire are removed from a reel for jobs as follows: for the first, 320 pounds; for the second, 492 pounds, for the third, 235 pounds. The lengths of wire unused are: $\frac{3}{64}$ of the first length, $\frac{1}{12}$ of the second length and $\frac{1}{5}$ of the third length. How much wire was actually used?

16. When full a barrel contains 1200 pounds of copper terminal lugs. If the barrel is $\frac{5}{8}$ full and each lug weighs $1\frac{11}{64}$ pounds, how many lugs are in the barrel?

CHAPTER III

DECIMAL FRACTIONS AND THEIR APPLICATIONS — THE CIRCLE

A decimal fraction is a fraction having for its denominator a number like 10, 100, 1,000, 10,000, etc., as for example,

$$\frac{5}{10}, \quad \frac{73}{100}, \quad \frac{951}{1000}.$$

A **decimal** is a fraction expressed without the denominator and with a period (.) — called the decimal point — placed *before* the numerator to show how the fraction should be read. Decimals are read in the same way as decimal fractions. Thus .67 is the same as $\frac{67}{100}$ and is read sixty-seven one-hundredths. The places at the right of the decimal point are called *decimal places.* The first place to the right is *tenths,* the second place is *hundredths,* etc. Figures to the left of the decimal point represent a whole number. For example 2.67 is read two and sixty-seven one-hundredths. A practical shop man would probably read 2.67 "Two-point-six-seven." Either method of reading decimals is correct.

The distance marked on the rule (Fig. 16) measures $2\frac{125}{1000}$ inches and is written as a decimal 2.125.

United States money is called decimal currency and is always written in decimal fractions of one dollar, thus one cent being $\frac{1}{100}$ of a dollar is usually written as a decimal with the ($) before the decimal point as $.01. Similarly, one dime, which is $\frac{1}{10}$ of a dollar, is written $.10; a quarter dollar is written $.25, and a half dollar is written $.50.

The ciphers following the decimals for the dime and half dollar need not, of course, be written as they have no significance. It is customary, however, to write them in this way for convenience in addition. Figures to the left of the

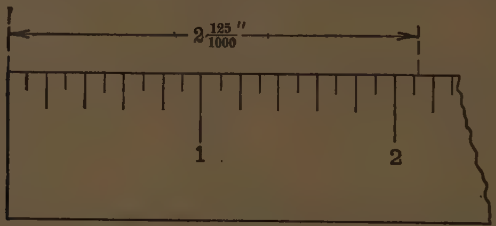


FIG. 16. Ruler marked in Decimal Fractions of Inches.

decimal point are whole numbers and in terms of money are, of course, dollars. Thus, if we write \$2.12 we mean two dollars, one dime and two cents, which we read as two dollars and twelve cents.

Keeping in mind the value of United States money makes it easier to remember that in any decimal system of numbers, as we advance toward the right from the decimal point, the value of each figure is $\frac{1}{10}$ of the figure at the left.

Principles of Decimals. There are certain principles which you should thoroughly understand if you are to work with decimals satisfactorily.

1. Moving the decimal point one place to the right, multiplies the decimal by 10; two places, multiplies by 100, etc. For, if the point is moved one place to the right, each figure will express ten times as much as before, hence the whole decimal will be ten times as great, etc.
2. Moving the decimal point one place to the left, divides the decimal by 10; two places, divides by 100, etc. For, if the point is moved one place to

the left, each figure will express 1 tenth of its previous value, hence the whole decimal will be only 1 tenth as great, etc.

3. Placing a cipher or zero between the decimal point and the decimal divides the decimal by 10. This moves each figure one place to the right; in which case each figure expresses 1 tenth as much as before; hence the decimal is only 1 tenth as great in value.
4. Annexing ciphers to the right of a decimal does not change its value, because each figure retains the same place as before, hence the value of the decimal is unchanged.*

The practical man must be able to read decimals from the scale (Fig. 17) and from the micrometer (Fig. 18). Moreover he should be able to write them as well as read them.

FIG. 17. Decimal Scale.

The smallest division on the scale in Fig. 17 is $\frac{1}{10}$ of an inch, the next larger division is $\frac{1}{5}$ of an inch. As decimals these would be written .1 and .2. Note that .2 is the same as $\frac{1}{5}$ $(.2 = \frac{2}{10} = \frac{1}{5})$.

Micrometer. Fig. 18 illustrates the micrometer caliper, a very interesting and efficient instrument for measuring un-

* A cipher, or zero, is sometimes written at the left of the decimal point to make the decimal plainer and avoid confusing the decimal point with some other marks; thus 0.25 is the same as .25 and 0.16 is the same as .16.

usually small distances, as for example the diameter of wire, rods, etc. With this instrument spaces in thousandths of an inch may be accurately measured.

The essential parts of a micrometer caliper are the stationary spindle A and the movable spindle B, which is threaded inside the stationary scale S with 40 threads to the inch. The thimble or cap T is fastened on the inside (at the milled end) to the movable spindle B. Note, therefore, that the spindle B and the thimble always move together,

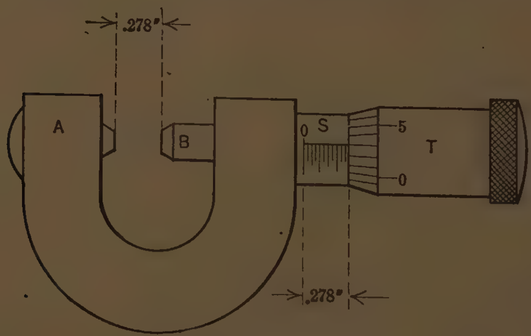


FIG. 18. Micrometer.

the latter over the scale S. Since there are 40 threads to the inch on the spindle B, one turn, or revolution, of the spindle will obviously change the position of the thimble $\frac{1}{40}$ inch, or .025 inch. It follows, therefore, that one turn of the thimble varies the opening between A and B $\frac{1}{40}$, or .025 inch. Each mark on the circular scale S represents one complete turn of the thimble or $\frac{1}{40}$ inch. Every fourth mark is extended and usually numbered 1, 2, 3, etc. Each of these extended marks represents a distance of $\frac{1}{10}$ inch between A and B. On the circumference of the thimble toward the scale S there are markings corresponding to 25 equal divisions. Therefore, turning the thimble

so as to move the edge of the thimble from one of these marks to the next turns the spindle B $\frac{1}{25}$ of a turn, or revolution, and moves the spindle $\frac{1}{25}$ of $\frac{1}{40}$, or $\frac{1}{1000}$ inch. Each of the small divisions on the circumference corresponds therefore to $\frac{1}{1000}$, or .001 inch. When the thimble of the micrometer caliper is at zero the spindles A and B should be just touching. In the position shown in the figure the thimble has been turned away from the zero point and the number of extended marks on the scale S show the number of tenths of inches in the reading. To get the complete reading, however, we must record (1) the number of tenths (as numbered); (2) the number of uncovered smaller divisions between the last uncovered "tenth" mark and the edge of the thimble; then multiply this number of divisions by .025 to make this reading in the decimal form; (3) the number read on the circular scale on the thimble opposite the horizontal line on S. This number is in thousandths of an inch and does not require multiplying. The complete reading of the micrometer gage showing the distance between the spindles A and B is then obtained by adding together the three readings mentioned above. This can best be illustrated by reading the micrometer with the thimble in the position shown in the figure with the following readings:

2 large divisions on the scale S	= .2	inch
3 small divisions between .2 mark and the edge of the thimble $= 3 \times .025$	= .075	"
Reading on the scale of the thimble	= .003	"
Total reading	.278	"

There is an increasing use of micrometer calipers by electricians and all kinds of metal workers, so that it is worth while for students in this course to understand how to read distances with it.

Reduction of Decimals to Common Fractions. The following examples show the methods of (1) reducing a decimal to a common fraction, and (2) reducing a common fraction to a decimal.

Example. Reduce .75 to a common fraction.

Solution. To express .75 as a fraction, we use 75 as the numerator and 100 as the denominator. Hence, $.75 = \frac{75}{100}$. $\frac{75}{100}$ may be reduced by dividing both numerator and denominator by 25. $\frac{75}{100}$, then, equals $\frac{3}{4}$.

Example. Reduce $0.16\frac{2}{3}$ to a common fraction.

Solution. $0.16\frac{2}{3}$ is read $16\frac{2}{3}$ hundredths. In expressing $0.16\frac{2}{3}$ as a fraction, the numerator is $16\frac{2}{3}$ and the denominator is 100. Therefore $16\frac{2}{3} = \dfrac{16\frac{2}{3}}{100}$. Change $16\frac{2}{3}$ to an improper fraction and divide it by 100. $16\frac{2}{3} \div 100 = \frac{50}{3} \div \frac{100}{1} = \frac{50}{3} \times \frac{1}{100} = \frac{50}{300}$, which, reduced to its lowest terms by dividing numerator and denominator by 50, equals $\frac{1}{6}$.

Example. Reduce $\frac{7}{8}$ to a decimal.

Solution. In any number you may annex ciphers after the decimal point without changing the value of the number. For example, $14.5 = 14.50 = 14.500 = 14.5000$, etc. Therefore in this expression $\frac{7}{8}$ the decimal point is placed immediately after the numerator 7. Then add a sufficient number of ciphers so that the division by 8 will come out even.

Calculation

$$\frac{7}{8} = \frac{7.000}{8} = 8\overline{)7.000}^{\,0.875}.$$

Addition and Subtraction of Decimals may be easily explained with a few examples.

Example. What is the sum of 45.37, .001, 56.508, 203, 75.45, .0007, and 86.497?

Solution. The first step in a problem like this is to write the numbers so that figures of the same order shall stand in the same column. This is easily done by placing the decimal points under one another. Now begin at the extreme right and add as in whole numbers.

```
 45.37
   .001
 56.508
203.
 75.45
   .0007
 86.497
--------
466.8267
```

Example. From 853.275 subtract 578.437.

Solution. Arrange the numbers exactly as for subtraction of whole numbers being careful to place the decimal points and figures of the same order (units, tens, hundreds, etc.), in the same column when numbers are thus arranged. Subtract as in whole numbers (see Chapter I).

Calculation A	Calculation B	Calculation C
853.275	1.001	5.999
578.437	.0002	3.8
274.838	1.0008	2.199

In calculations like B and C until thoroughly practiced add ciphers in arranging work as follows:

1.0010	5.999
.0002	3.800
1.0008	2.199

Multiplication and Division of Decimals is illustrated by the following examples:

Example. Multiply 4.23 by .36.

Solution. Arrange your figures as you do in ordinary multiplication. Then multiply as in whole numbers. In placing the decimal point in your answers point off as many places from right to left as you have decimal places in both the multiplied numbers.

Calculation

```
  4.23
   .36
------
  2538
 1269
------
1.5228
```

You may test the accuracy of this method by changing the decimals to common fractions and multiplying them together, thus:

$$4.23 \times .36 = 4\tfrac{23}{100} \times \tfrac{36}{100} = \tfrac{423}{100} \times \tfrac{36}{100} = \tfrac{15228}{10000} = 1\tfrac{5228}{10000} = 1.5228.$$

Briefly the method is as follows: Multiply as in whole numbers, and from the right of the result point off as many decimal places as there are in both numbers.

Example. Divide 272.636 by 6.37.

Solution. First, divide as you would in whole numbers. To find the number of decimal places in your answer, subtract the number of decimal places in the divisor from the number of decimal places in the number to be divided, called the dividend. This result will give you the number of decimal places in the answer, or quotient. In the number 272.636 there are three decimal places. In the divisor 6.37 there are two decimal places. Therefore in the quotient you will have one decimal place counting from the right, as follows: 42.8.

Calculation

```
          42.8
6.37 )272.636
      2548
       1783
       1274
        5096
        5096
```

At this point other suggestions on division of decimals may be helpful. Arrangement of work is always important in mathematics, but nowhere more important than in decimals. Unless your work is clearly arranged you are likely to become confused. In the division of decimals, the arrangement of divisor, dividend, and quotient, as shown in the example above, is strongly recommended. With this arrangement many students have found it easy to place the decimal point.

Many consider it more simple, before dividing, to change the divisor, if a decimal, to a whole number by moving the decimal point the necessary number of places to the right. If you use this method be sure to move the decimal point in the dividend the same number of places to the right, adding ciphers for filling if necessary. This method is illus-

trated in preceding problems. Study the following arrangements for an illustration of this method:

Example. Divide .819 by .39.

Arranged for solution: $39\overline{)81.9}$

Example. Divide .39 by .819.

Arranged for solution: $819\overline{)390.00}$

Example. Divide 7.56 by .002.

Arranged for solution: $2\overline{)7560.}$

Example. Divide 152 by .76.

Arranged for solution: $76\overline{)15200.}$

Many students have found it less confusing to place the decimal point in the proper position before dividing.

Specimen Problems Solved and Explained

Example. Divide .1995 by 15.

Solution.

$$\begin{array}{r} .0133 \\ 15\overline{).1995} \\ \underline{15} \\ 49 \\ \underline{45} \\ 45 \\ \underline{45} \\ 0 \end{array}$$

In this example you will note that there are *no* decimal places in the divisor and 4 decimal places in the number to be divided. Thus there are 4 minus 0, or 4 decimal places in the quotient. But by actual division there are only 3 figures in the quotient, namely 133. It is necessary, therefore, to prefix one cipher to the result of division. This gives the correct answer .0133. Sometimes it is necessary to prefix a larger number of ciphers, as

would be the case if we should divide .0045 by 22.5; then the result would be .0002.

Solution.

$$\begin{array}{rl} & .0002 \\ 22.5 & \overline{).00450} \\ & \underline{450} \end{array}$$

Example. Divide 6.336 by .09.

Solution.

$$\begin{array}{rl} & 70.4 \\ .09 & \overline{)6.336} \\ & \underline{6\,3} \\ & 3 \\ & \underline{0} \\ & 36 \\ & \underline{36} \end{array}$$

or changing the divisor to a whole number we have

$$\begin{array}{rl} & 70.4 \\ 9 & \overline{)633.6} \\ & \underline{63} \\ & 3 \\ & \underline{0} \\ & 36 \\ & \underline{36} \end{array}$$

In both solutions there is one more decimal place in the number to be divided than in the divisor. Therefore, one decimal place is pointed off in the quotient.

Example. Divide .486 by .003.

Solution.

$$\begin{array}{rl} & 162 \\ .003 & \overline{).486} \end{array}$$

or changing the divisor to a whole number we have

$$\begin{array}{rl} & 162 \\ 3 & \overline{)486} \end{array}$$

In this example you have the same number of decimal places in the number to be divided as in the divisor; therefore, there are no decimal places in the quotient; that is, the quotient is a whole number.

Example. Divide 981 by .75

Solution.

$$
\begin{array}{r@{\,}l}
 & 1308 \text{ quotient and answer} \\
.75 & \overline{)981.00} \\
 & \underline{75} \\
 & 231 \\
 & \underline{225} \\
 & 600 \\
 & \underline{600}
\end{array}
$$

or changing the divisor to a whole number we have

$$
\begin{array}{r@{\,}l}
 & 1308. \\
75 & \overline{)98100.}
\end{array}
$$

Here we had to add two ciphers to the number to be divided equal to the number of decimal places in the divisor. As the number to be divided contains the divisor without remainder the quotient is a whole number.

It frequently happens in solutions similar to the last that the divisor is not contained in the number to be divided an exact number of times. In that case add ciphers to the number to be divided and continue dividing until the desired number of decimal places has been reached.

It is rarely necessary to obtain an answer carried farther than the fourth decimal place. Usually an answer of two or three decimal places is sufficient. To bring such solutions to a close determine to how many decimal places the divisor is to be carried and carry your work one place further. If the last figure of the quotient is 5 or more, add 1 to the preceding figure and write after it the minus sign (−). If the figure so secured is less than 5 omit it and write the plus sign (+) in its place. If some method like the above were not used divisions would never terminate. According to this rule,

3.1416 might be written 3.142−
3.141 " " " 3.14+

The number of places to which such divisions should be carried depends entirely on the desired accuracy of calculations.

Illustrative Problems

Example. An iron bar in Fig. 19 is 9.21 inches long by 2.47 inches wide by .37 inch thick. Find its weight if a cubic inch of iron weighs .261 pound.

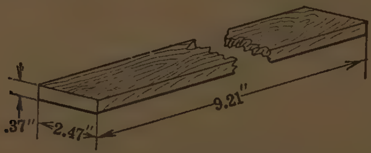


Fig. 19. Dimensioned Bar.

Solution. The contents of a rectangular solid such as this is obtained by multiplying together length, breadth and thickness $9.21 \times 2.47 \times .37 = 8.417$ cubic inches.

As each cubic inch weighs .261 pounds the total weight of the bar will be 8.417 times .261 pounds, or 2.196 pounds.

Circle. The rim of a circle (Fig. 20), is its *circumference*, all points of which are equally distant from its center.

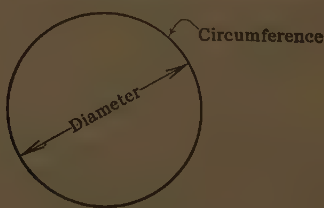


Fig. 20. Circle.

The *diameter* of a circle is any straight line passing through the center and touching the circumference at two points.

A *radius* of a circle is any straight line from center to circumference. A radius is always exactly one half the diameter. The

plural of radius is radii. We say "one radius" but "several radii." Roughly speaking the spoke of a wheel is a radius.

The circumference of any circle is 3.1416 *times as long as its diameter and* 6.2832 *times as long as its radius.*

The rules for the circle apply to wheels, pulleys, ends of cylinders, gears, etc.

The following examples illustrate calculations in which the above explanations of circles are applied.

Example. In Fig. 21 a brass plate is represented in which two holes are drilled. Find the length of space between the holes.

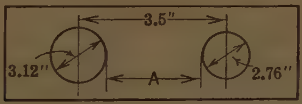


FIG. 21. Dimensioned Plate with Circles.

In this problem the radius of the left hand hole will be one half of 3.12 inches (1.56 inches), the radius of the hole at the right will be one half of 2.76 inches (1.38 inches). If we add 1.56 inches and 1.38 inches and subtract the result from 3.5 inches, the desired result A should be obtained.

$$1.56 \text{ inches} + 1.38 \text{ inches} = 2.94 \text{ inches}$$
$$3.5 \text{ inches} - 2.94 \text{ inches} = .56 \text{ inches.}$$

Note that the sketch in Fig. 21 is not drawn according to dimensions. This is not unusual in freehand drawings and should not confuse you.

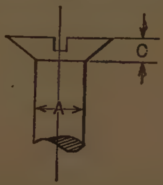


FIG. 22. Screw Head.

Example. In a standard screw (Fig. 22), the dimension C has a direct relation to the dimension A. This is represented by the formula, $C = \dfrac{A - .0052}{1.739}$.

Solution. To obtain the value of C for any given value of A, we subtract .0052 from the value of A and divide the result by

1.739. We shall assume that A in this case equals $\frac{7}{8}$ inch. Reducing, $\frac{7}{8}$ to a decimal, we find that $\frac{7}{8}$ of an inch equals .875 inch.

$$.875 - .0052 = .8698$$
$$.8698 \div 1.739 = .100 \text{ inch} = C.$$

Problems — Group I

1. A man owning 17.63 acres of land sold $1\frac{1}{8}$ acres to one person and $\frac{1}{40}$ of an acre to another. How much land did he have left?

2. A carpenter paid $\$27\frac{5}{8}$ for a mantel, \$33.40 for a grate and $\$9\frac{3}{25}$ for a hearth. How much did he pay in all?

3. One quart liquid measure contains 57.75 cubic inches and one quart dry measure contains 67.2 cubic inches. How many cubic inches larger is the dry quart than the liquid quart?

4. What is the distance around (circumference) a cart wheel if its diameter is 3.5 feet?

5. A pipe used as a conduit for electric wires has the dimensions indicated in Fig. 23. How thick is the pipe?

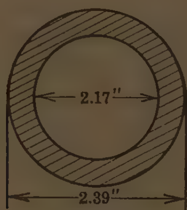


Fig. 23. Cross-section of Conduit.

6. What will it cost to lay 5.4 miles of steel car track if the rails weigh $43\frac{1}{3}$ pounds per foot and cost \$48.50 per ton?

7. A cubic foot of water weighs 62.5 lbs. Find the weight of a cubic inch of water (one cubic foot contains 1728 cubic inches).

8. A $\frac{3}{4}$-inch bore when checked by the inspector is found to measure .761 inches. How much oversize is it?

9. If a steel tape (Fig. 24) expands .00016 inch for every inch of length when heated to a certain temperature, how much will a tape 100 feet long expand?

10. A wire 9 feet 4 inches long was suspended with a weight of 325 pounds (Fig. 25) on the end of it. It was found that this

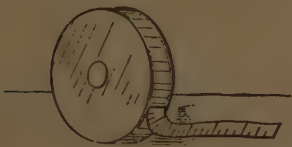


FIG. 24. Measuring Tape and Case.

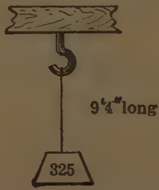


FIG. 25. Suspended Weight.

weight stretched it .265 inch. What was the average stretch per foot?

11. How much must be paid for 1600 feet of six-cornered steel bars (Fig. 26) if their weight is 1.87 pounds per linear foot and

FIG. 26. Six-cornered Steel Bar.

the price is $46.50 per hundred pounds?

12. Estimates of building construction costs are often approximately made by figuring the cubic feet. What would be the approximate cost of constructing a power plant whose cubic contents was 273,546 cubic feet if the cost of construction was figured at 23 cents per cubic foot?

13. If the *tread* on a certain stairway (Fig. 27) is 1.6 times as wide as the *riser*, how many inches high will the riser be if the tread is 9 inches wide?

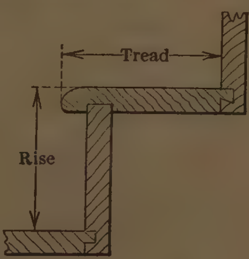


FIG. 27. Stairway Construction.

14. If the distance from the level of the first floor to the second floor level is 9 feet, 3.3 inches, how many steps will be necessary to connect one floor with the other if the riser of each step is 5.3 inches high?

15. An electric cable (Fig. 28) has a diameter of .23 inches not including the insulation. Assuming that the insulation is .07 of an inch thick and that the wire runs through an insulator which has an outside diameter of .65, what size hole should be made to accommodate the wire allowing .03 of an inch on each side of the wire for an "easy" fit?

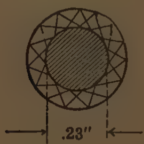


FIG. 28. Insulated Electric Cable.

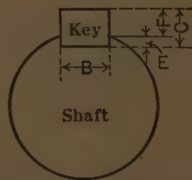


FIG. 29. Shaft and Key.

16. If the dimension B of the key for a shaft in Fig. 29 is 1.3 times that of C and C is .35 inches, what is the dimension of B? If E is $\frac{1}{4}$ as much as F, what are the dimensions of E and F?

Suggestion. $E + F = C = .35$, but $E = \frac{1}{4}F$, therefore, $E + F = \frac{1}{4}F + \frac{4}{4}F = \frac{5}{4}F = .35$. Solve for F.

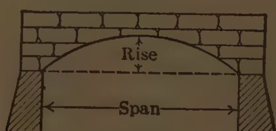


FIG. 30. A Masonry Arch, showing Rise and Span.

17. The "rise" of an arch (Fig. 30) is .17 of the "span." How many times greater is the span than the rise?

FIG. 31. Standard Nut for a Bolt.

18. On a standard nut (Fig. 31) the distance A (generally called the distance across flats) in the figure is equal to one and one half

times the diameter of the bolt plus one eighth of an inch. What would the distance A be if the bolt had a diameter of 1 inch? of $1\frac{1}{4}$ inches? (Remember that $\frac{1}{8}$ of an inch expressed as a decimal is .125 inch.)

19. A steel rod will stand a "pull" of 125,000 pounds before heating. If the area of a rod cross-section is 2.5 square inches, what is the pull per square inch?

20. The total area of the outside walls of a house is 3700 square feet including the windows. If there are 18 window openings in the house and each window opening contains 17.5 square feet, what part of the total area are the window openings? *ABCD* is the window opening (Fig. 32).

FIG. 32.
Window Opening.

PROBLEMS — GROUP II

1. Add six dollars and seventy-five cents, three dollars and fifty-one cents, two dollars and three cents, and seven dollars.

2. An electric motor was purchased for \$75.25 and sold for \$100. What was the profit?

3. A flat copper bar to be used behind a switchboard is required to be exactly .500 inch wide. When measured by micrometer caliper the width is found to be .499 inch. How much too small is it?

If you have never used a micrometer caliper arrange for some instruction with a friend who owns one. If there is a machine shop in your town or city such an arrangement should not be difficult.

4. Because of an error in adding fractions a generator armature was furnished with a shaft 1.25 inches in diameter and the pulley intended to fit this shaft was made with a hole 1.375 inches in diameter. How much too large is this hole?

5. A screw used for fastening a switch to a wall passed through the porcelain switch base .375 inch thick, plaster .125 inch thick, and entered a wooden support .5625 inch. What was the length of the screw?

6. What is the total weight of 178 fuses weighing 0.566 pounds each?

7. How many square feet of floor space are occupied by a steam turbine and alternator which requires .9018 of the total floor area of 2250 square feet.

8. A tank holds 24,051 pints of insulating varnish. If .796 pint weighs 1 pound, how many pounds of varnish are there in the tank?

9. In an induction motor the rotating part which is called the rotor is surrounded by a stationary part called the stator. In order that the rotor may revolve freely the hole in the stator must be somewhat larger than the rotor. This leaves what is known as an air-gap between the rotor and stator. If the hole in the stator is 24.375 inch in diameter and the diameter of the rotor is 24 inches, what is the width of the air gap?

10. A pile of scrap insulated copper wire weighs 166,250 pounds. If $\frac{4}{10}$ of the weight is due to insulation, what is the weight of the bare copper wire?

11. The distance between centers of two holes in a copper switch blade is $3\frac{1}{2}$ inches. If the holes are $\frac{15}{16}$ of an inch in diameter, what length of metal is left between them?

12. In wiring for electric bells which are rung by means of two or three dry batteries, the wires may be placed very close together. If $\frac{3}{8}$ of an inch is allowed for each wire, how many wires can be placed in a space 7 feet 6 inches wide?

13. A dwelling house can be wired by 5 men in 6 days. How long would it take if 8 men were put on the job?

14. A layer of wire is wound on a pipe $15\frac{9}{32}$ inches long. The wire has a diameter of .181 inch. How many turns of wire can be wound on the pipe?

CHAPTER IV

PERCENTAGE — PRACTICALLY APPLIED

Almost every day you hear that Mr. Smith made a profit of 20 per cent on his automobile business, or that the amount of steel in one ship is 50 per cent of that in another of different type. You will remember that a decimal fraction always has for its denominator 10, 100, 1000, 10,000, etc. Percentage relates to calculations which involve the use of decimal fractions having for their denominator the number 100.

Percentage is derived from the term per cent, whose symbol is % and means *on the basis of one hundred.* It therefore offers a convenient method of calculation. As the denominator is always 100, it is easy to add, subtract, multiply and divide the fractions used. For instance, it is much less confusing to say $.46, or $\frac{46}{100}$ of a dollar, than to say $\frac{69}{150}$ of a dollar or $\frac{23}{50}$ of a dollar, though all three fractions have exactly the same value.

In percentage operations we always work on the understanding that quantities are divided into 100 parts. Thus 6 per cent of a number means $\frac{6}{100}$ or .06 of that number. For example, 6 per cent of 150 = $150 \times \frac{6}{100} = 150 \times .06 = 9$0.

Common per cents and their decimal equivalents are given in the table below:

Per cent	Decimal	Per Cent	Decimal
1	.01	$33\frac{1}{3}$	$.33\frac{1}{3}$
$1\frac{1}{2}$	.015	$87\frac{1}{2}$	.875
3	.03	100	1.00
5	.05	150	1.50
10	.10	300	3.00
20	.20	500	5.00

The terms used in percentage problems are *base, rate, percentage,* and *amount.*

The *base* is the number or quantity which is understood to be divided into 100 equal parts. Example: In the year 1910, Springfield, Massachusetts, had approximately 89,000 inhabitants; in 1920 the number had increased 44 per cent. Here 89,000 is the base.

The *rate* is the number of the 100 equal parts which are considered. Example: I saved $45, which is one year's interest on $900 which I had invested at 5 per cent. Here 5 per cent is the rate.

Percentage is the quantity obtained by multiplying the *base* by the *rate.* Thus in the last example 5 per cent of $900 is 900 × .05 = $45.00 (the percentage).

The *amount* is the total of base plus percentage. Example: My savings amounting to $1000, with the addition of a year's interest at 4 per cent, totaled $1040. Here $1040 is the *amount.*

To fix the subject more clearly in mind we shall work out the solutions of various common percentage problems.

Example. From a coal pile containing 755 tons 20 per cent was carted away? What percentage was removed?

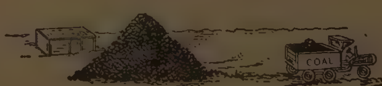


FIG. 33. Problem in Percentage.

Solution. Here the base is 755, that is, the quantity from which a certain number of 100 equal parts are to be taken; the rate is .20 ($\frac{20}{100}$) or the number of hundredths to be taken. Therefore the percentage removed was: $755 \times \frac{20}{100} = 755 \times .20 = 151$ tons.

A short way of expressing this method of finding percentage is as follows:

$$\textit{Percentage} = \textit{Base} \times \textit{Rate}.$$

Example. Twenty per cent of a pile of coal was carted away. If 151 tons were removed how many tons were in the pile?

Solution. If 151 is 20 per cent of a certain quantity, then 1 per cent of that quantity would be $\frac{1}{20}$ of 151 or $151 \div 20 = 7.55$. Then, if 7.55 is one per cent, the entire quantity, 100 per cent, would be $7.55 \times 100 = 755$ tons.

In this solution we might have avoided the extra work of multiplying by 100, if we had changed $\frac{20}{100}$ to the decimal 0.20 and divided, thus: $151 \div 0.20 = 755$.

Now turning back to the original problem we find that 151 (the percentage) $\div$ .20 (rate) = 755 (base).

You will have to remember that

$$Base = Percentage \div Rate.$$

Example. From a coal pile containing 755 tons, 151 tons were carted away. What per cent of the entire pile was removed?

Solution. The base is 755; 151 represents the percentage, the portion removed. Here 1 per cent of 755 is $755 \times .01 = 7.55$. Therefore 7.55 being 1 per cent, 151 contains as many per cents as 7.55 is contained in 151.

$$151 \div 7.55 = 20 \text{ per cent.}$$

Another way of securing the same result is as follows:

The coal pile originally contained 755 tons; 151 of these tons were hauled off. That is, one hundred fifty-one seven hundred fifty-fifths ($\frac{151}{755}$) was removed.

Reducing the common fraction $\frac{151}{755}$ to a decimal we have $151 \div 755 = .20 = 20$ per cent.

$$20 \text{ per cent (rate)} = 151 \text{ (percentage)} \div 755 \text{ (base).}$$

A short, easily remembered way of expressing all this work is

$$Rate = Percentage \div Base.$$

Example. A coal pile originally containing 755 tons is increased by 20 per cent. How many tons were in the pile when the extra coal was added?

Solution. This problem illustrates the method of finding the *amount.* The *original* quantity, or 755 tons, is the base, and .20 is the rate. 755 (base) $\times$ .20 (rate) = 151, the percentage. The problem calls for the addition of the percentage to the base or 755 (base) + 151 (percentage) = 906 (amount). Therefore amount − base = percentage. As the base is always present in the amount we can simplify the method of securing the amount by adding 1 to the rate and multiplying the base by this sum. For instance, 755, which is the base, times one and twenty hundredths (which is the 1 referred to above plus the rate) equals 906, the amount. In condensed language we say

$$Amount = Base \times (1 + Rate).$$

The amount shows an increase or gain in the base, or original quantity.

In the foregoing four examples we have illustrations of all the important principles used in the solution of percentage problems. Do not be deceived by the language of the problem. Obviously percentages apply to other quantities as well as to the piles of coal which have been used in the above examples. In the practical work of the shop, percentage problems are involved in gain and loss of the power and speed of machinery, depreciation of equipment, the rise and fall of wages, and the like.

Many students find it difficult to recognize the base. Of course, there is no strict rule that we can say must be followed at all times, but it may help you to remember that the *base* is the original quantity or number upon which a percentage is to be figured; the number with which we wish to compare some new or changed condition. If we are reckoning profits we have to compare the selling price with the *original cost* price. If we are to determine the efficiency of a motor, an engine, or a machine, we must compare the work it actually does with its *original rated capacity.* If a machine, engine, motor, investment, purchase, contract, or

the like, produces results according to original specifications, it is regarded as 100 per cent efficient. Any change from original conditions, be it in speed, power, weight, price or other element, is expressed as percentage either over or under.

List Prices and Discounts. List prices often give no idea of actual selling prices. Discounts may be expressed "fifty and ten off." This does not mean that the discount is 60 per cent of the list price, but rather that a discount of 50 per cent is first made, and then a discount of 10 per cent is made on the remainder.

If the list price of a certain article is \$4.50, and the discount is "50 and 10 off," the actual price to the purchaser would be found by first taking 50 per cent of \$4.50 and subtracting it from \$4.50; then taking 10 per cent of the remainder and subtracting it from the remainder:

50 per cent of \$4.50	= \$2.25	— First Discount
\$4.50 – \$2.25	= \$2.25	— After deducting First Discount
10 per cent of \$2.25	= \$ .225	— Second Discount
\$2.25 – \$.225	= \$2.025	— Final price.

Illustrative Problems on Percentage

Example 1. A man deposited \$1250 in the bank; he drew out 15 per cent of it the first month, 20 per cent of the remainder the next month, and having realized $18\frac{3}{4}$ per cent on what he had drawn, deposited it. What was his bank deposit then?

Solution.

15 per cent of \$1250 is \$187.50

\$1250 – \$187.50 = \$1062.50 (what was left after first withdrawal)

20 per cent of \$1062.50 = \$212.50.

\$1062.50 – \$212.50 = \$850.00 (what was left after second withdrawal).

He has now drawn \$400 in all. If this is multiplied by $.18\frac{3}{4}$, the result obtained will be the amount of his new deposit.

\$400 times $.18\frac{3}{4} = \$400 \times .1875 = \$75.00$.

$\$850 + 75 = \$925$, his final balance in the bank.

Example. The revolutions per minute of a certain flywheel (Fig. 34) are 450. If the speed is increased to such an extent that the speed is 525 revolutions per minute, what is the percentage of increase?

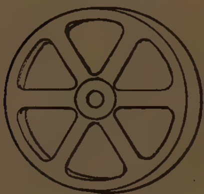


Fig. 34. Fly Wheel.

Solution. The increase in this case is 525 − 450, or 75 revolutions per minute. The per cent of increase will be $\frac{75}{450} = .16\frac{2}{3} = 16\frac{2}{3}$ per cent.

Example. A certain alloy contains 9 parts of copper, 3 parts of tin, and 2 parts of zinc. (*a*) In a block of this alloy weighing 26.88 pounds, how many pounds of each will there be? (*b*) What per cent of the alloy does each metal represent?

Solution. It is evident that there are 14 parts in all. Each part may be thought of as weighing 26.88 pounds $\div$ 14 = 1.92 pounds.

9×1.92 lbs. = 17.28 lbs. of copper.
3×1.92 lbs. = 5.76 lbs. of tin.
2×1.92 lbs. = 3.84 lbs. of zinc.

$\frac{17.28}{2.688}$ = per cent of copper = 64.3 per cent.

$\frac{5.76}{26.88}$ = per cent of tin = 21.4 per cent.

$\frac{3.84}{26.88}$ = per cent of zinc = 14.3 per cent.

Example. The grade of an incline is expressed in per cent. That is, a rise of 1 foot in 100 feet of horizontal distance would be a 1 per cent grade; a rise of $2\frac{1}{2}$ feet in 100 feet of horizontal distance would be a $2\frac{1}{2}$ per cent grade, etc. If the difference between the bottom and the top of a $1\frac{1}{4}$ per cent grade is 56.2 feet (Fig. 35), what is the horizontal length necessary to obtain this grade?

Solution. Since for every 100 feet of horizontal distance the rise is $1\frac{1}{4}$ feet, we can get the number of 100-foot lengths correspond-

ing to the total rise of 56.2 feet by finding the number of times 56.2 is larger than 1.25. There are then, $\frac{56.2}{1.25} = 44.96$ hundred-foot units of horizontal length; or the total horizontal length is $44.96 \times 100 = 4496$ feet.

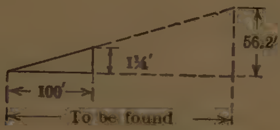


FIG. 35. Percentage of Grade.

Summary. From your study of this chapter you should have learned the following:

1. First of all, that percentage is the process of solving problems in which 100 is taken as the basis.
2. The second point brought out was the discussion of the four terms used in percentage, viz., *base*, *rate*, *percentage*, and *amount*. Can you explain and illustrate each of these?
3. Then the important principles used in solving the various kinds of percentage problems were given.

   These are as follows:

   (*a*) Percentage = Base × Rate
   (*b*) Base = Percentage ÷ Rate
   (*c*) Rate = Percentage ÷ Base
   (*d*) Amount = Base × (1 + Rate)
4. The last topic discussed in this assignment was "List Prices and Discounts." Remember that a discount of 20 per cent and 10 per cent does not mean a total discount of 30 per cent, but means a discount first of 20 per cent and then a discount of 10 per cent on the remainder.

Problems — Group I

1. After the wages of a workman were reduced $8\frac{1}{2}$ per cent he received \$5.70 per day. What were his wages before reduction?

2. An electric motor cost \$750 delivered. For what must it be sold to gain $33\frac{1}{3}$ per cent?

3. A man bought some stock in an electric lighting company and a year later sold it for \$140 per share. He gained 27 per cent on the transaction. What was the price paid per share?

4. An equalizing tank has a capacity of 162 gallons. If there is a constant flow into the tank of 72 gallons per hour and a discharge of 80 per cent of the amount flowing into the tank, in what time will the tank be filled?

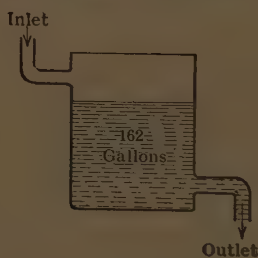


Fig. 36. Equalizing Tank.

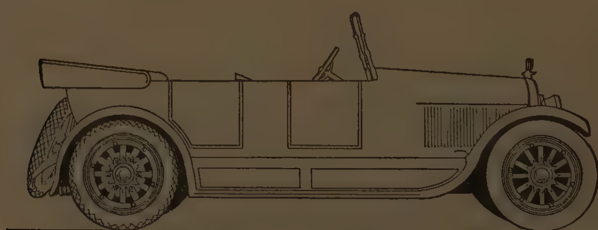


Fig. 37. Automobile.

5. A man bought a used automobile for \$550. He used it a year and turned it in for a new one. If the depreciation in value was figured at $12\frac{1}{2}$ per cent what was he allowed on the new car?

6. An automobile owner purchased four tires for his car 20 per cent and 10 per cent off list price with a further discount of 2 per cent for cash. The list price of each tire was $36.90. How much did he pay for the four tires?

Fig. 38. Wire Nail.

7. If wire nails (Fig. 38) cost 7 cents per pound in 1912 and 18 cents per pound in 1920, what was the per cent of increase in price?

8. A certain brand of spark plug retails at $1.50. These may be purchased at a reduction of 15 per cent if purchased in quantity. How much will a gross of them cost if the reduction is allowed on this amount? (A gross equals 12 dozen).

9. The actual cost of excavating the rock on a job is $.70 per cubic foot. If 12 per cent is added for overhead expenses and 10 per cent is added for profit what will the owner have to pay per cubic yard for the actual excavation?

10. What is the rise per foot in the gable roof illustrated in Fig. 39? What is the per cent of rise per foot of width?

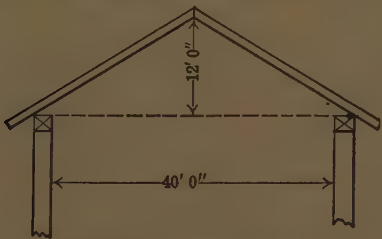


Fig. 39. Gable Roof.

11. A man claims that his total return on an apartment house investment is only 10 per cent. The house cost him $30,000. The taxes are $22.30 per $1,000 of the cost, per year, the insurance is figured at the rate of $8.00 per $1000 of the cost for *five* years, the depreciation is computed at 5 per cent of the cost per year, the water rates and other expenses are figured at 5 per cent of the cost. There are 10 apartments in the house. He charges $90 per month rent for each apartment. What is the *net* return on the investment?

12. A man buys 10 tons of coal for which he pays $13.50 per ton. Upon examination he finds that about 12 per cent of it is slate. What should the total bill be, in justice to him, figuring the slate worth nothing?

13. If a new touring car cost $2190 delivered and the depreciation the first year was 25 per cent, the second year 10 per cent of the value at the end of the first year, the third year 5 per cent of the value at the end of the second year, what would it be worth at the end of the third year?

14. The specific gravity of a block of wood is such that when it floats in water, 0.62 of it is submerged. What per cent of it is above the surface of the water?

FIG. 40. Specific Gravity Problem.

15. If a barrel of crude oil (Fig. 41) costs $4.20 and it is found upon receipt that $3\frac{1}{2}$ per cent of it has leaked out, what should the buyer pay for the barrel?

FIG. 41. Leakage Problem.

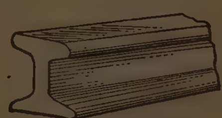


FIG. 42. Steel I-beam.

16. A steel I-beam exposed to the sun's rays expands $\frac{1}{100}$ of 1 per cent of its length. If it is $22\frac{1}{6}$ inches long before exposure what will the total expansion be?

17. If I sold a motor cycle for $275 and thereby gained 12 per cent of the cost, what did I pay for it?

18. Water weighs 62.5 pounds per cubic foot. How much will one gallon of water weigh if there are 231 cubic inches in a gallon? What per cent of a cubic foot in a gallon? (See page 9).

19. The sales of a book on Electricity for 1917 were 5260; for 1918, 9700; for 1919, 12,425. (See Fig. 43). What was the per cent of increase each year?

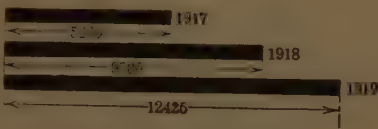


Fig. 43. Diagram of Sales.

20. If the revolutions per minute of a commutator of a large electric generator (Fig. 44) increase uniformly for 4 minutes and at the rate of 15 per cent over the preceding minute, what will the

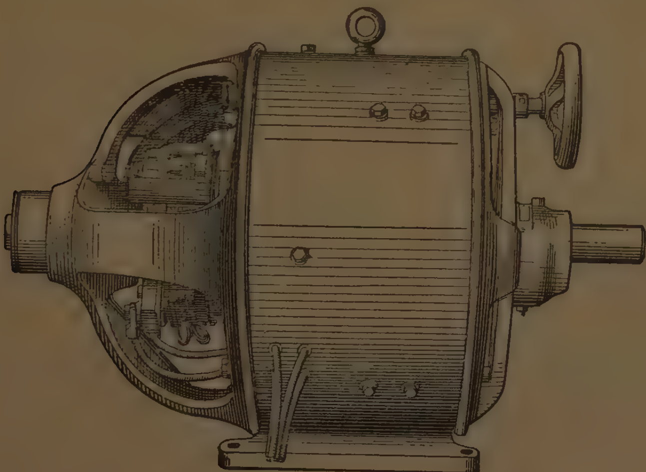


Fig. 44. Electric Generator.

speed be at the end of four minutes if the speed at the end of the first minute was 900 revolutions?

Problems — Group II

1. The valuation of the tools and machinery in a power plant is $1,605,835.84. If 6 per cent per year is allowed for depreciation and replacement, what would this allowance be?

2. An ordinary lighting transformer consists of two coils of wire wound on an iron core. This iron core is made of thin pieces of sheet steel which are insulated from each other by coating each with an insulating compound. If the transformer core has a total volume of 879.83 cubic inches and 6.5 per cent is allowed for insulation, how many cubic inches of actual steel are there in the core?

3. The copper wire on an armature weighs 64 pounds, 10 ounces, which is 5.5 per cent of the total weight of the armature. How much does the armature weigh?

4. Most electric cars are equipped with four motors. The combined weight of four motors of one type of car is 10,196.73 pounds, which is 9.5 per cent of the total weight of the car. What is the total weight?

5. The wooden poles of a street railway system have a life of 7 years if untreated. This is 60.34 per cent of the life of these poles when treated with creosote. What is the life of a treated pole?

6. A company's pay roll is $443,210.69 which is 23 per cent of the total receipts. What are the receipts of this company?

7. A motor ran slowly because of low voltage. When this was remedied it delivered 4312.5 horse power, which was 15 per cent more than before. How many horse power were delivered originally?

8. The temperature of electrical apparatus depends upon the amount of surface available for radiation. In order to increase this radiating surface spaces are often left when assembling the armature cores of motors and generators. The spaces are known as ventilating ducts. If by using ventilating ducts the radiating area of a motor is 735 square inches, which is an increase of 47 per cent, what is the area without ducts?

9. It is found desirable to mount a motor directly on each machine manufactured by a certain company, giving what is known

as "individual drive." The weight of the entire machine is then 1475 pounds, an increase of 18 per cent. What is the weight of the machine without the motor?

10. An electrician uses 4646.4 feet of wire in wiring a house. The remainder of the wire, which is 12 per cent of the total amount, is used for wiring the garage. What is the total length of wire used?

11. A turbine generator weighs 77,350 pounds. Of this amount 50,775 pounds is the weight of the stator (stationary part) of the electric generator and the remainder of the weight is in the rotor (rotating part). What per cent of the total weight is in the stator?

12. The labor item in the following bill is what per cent of the total amount?

Labor...........................	$49.20
Material........................	$ 9.43
Incidentals.....................	$ 1.75
Traveling Expense	$ 1.12

13. In testing out 2880 fuses it was found that 219 were "blown" and could not be used again. What per cent of the fuses were "blown"?

14. The pieces of sheet steel used in the cores of transformers and armatures are punched out of large sheets. If a sheet contained 56 square feet before punching and 14 square feet are punched out, what per cent of the sheet is punched out? What per cent is left?

15. An electrician contracted for a wiring job for $380. This was found to be 76 per cent of the actual cost of the job. How much did he lose on the contract?

16. At what per cent above or below rated speed is each of the following motors operating? (r.p.m. = revolutions per minute.)

Motor	Rated speed	Actual speed
	r.p.m.	r.p.m.
A	1212	1220
B	1195	1178
C	1650	1635
D	1780	1809

17. An incandescent lamp rated at 48 candle power is operated on a voltage higher than that specified on the label with the result that for a short time it gives 192 candle power and then burns out. What per cent of this latter candle power is the rated value?

CHAPTER V

RATIO AND PROPORTION

Ratio is another name for division. Thus the ratio 2 to 4 is the same as 2 divided by 4. When the numbers 2 and 4 are written as a ratio the form used is 2 : 4; when written as a division the form 2 ÷ 4 is used. The difference between these two written forms is only in the omission of the small horizontal line in writing the ratio. Both have exactly the same meaning. Since division can always be expressed also as a fraction we shall often see the ratio of two numbers written as a fraction. The ratio of 2 to 4 can be written in the following three ways which have equal values, thus, 2 : 4; $\frac{2}{4}$; and 2 ÷ 4.

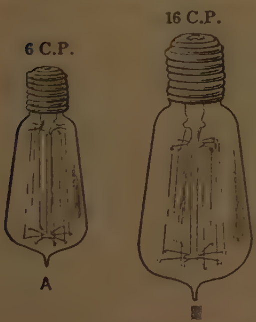


FIG. 45. Electric Lamps.

Whether a ratio is expressed as a fraction or in the form 2 : 4, it is read "2 is to 4" or "the ratio of 2 to 4." A practical example of the meaning of ratio will help to an understanding of this subject. Two electric lamps are shown in Fig. 45. The larger is for 16 candle power, and the smaller for 6 candle power. Then the ratio of the amount of light from the larger lamp is to the amount of light from the smaller as 16 is to 6. This means that the larger gives $\frac{16}{6}$ or $2\frac{2}{3}$ times as much light as the smaller. We can just as correctly say this the other way around, that the ratio of the amount of light from the smaller lamp is to that from the larger as 6 is to 16, which means that

the smaller lamp gives only $\frac{6}{16}$ or $\frac{3}{8}$ as much light as the larger one.

Fig. 46 shows two pulleys, M and N, and a connecting belt. If the pulley M runs at a speed of 125 revolutions per minute

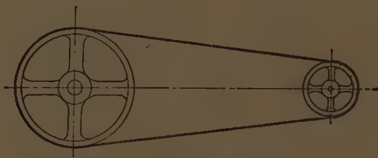


FIG. 46. Belted Pulleys.

and the pulley N, to which it is connected, runs at a speed of 300 revolutions per minute, the ratio of the speed of M to the speed of N is as 125 is to 300 or 125 : 300, which, as a fraction, can be written $\frac{125}{300} = \frac{5}{12}$.

Proportion. It is important to understand the meaning of proportion as used in mathematics. The words "Ratio and Proportion" are almost invariably used together. A proportion shows that *two ratios are equal*, as $1 : 2 = 2 : 4$.

Formerly the equality of numbers was indicated most frequently by the sign (::). In recent years the regular sign of equality (=) has gained favor. The use of both signs is permissible and generally understood. For example $9 : 18 :: 2 : 4$ is the same as $9 : 18 = 2 : 4$. This proportion may also be written $9 \div 18 = 2 \div 4$; or $\frac{9}{18} = \frac{2}{4}$. It may be read either

9 is to 18 as 2 is to 4

or

the ratio of 9 to 18 equals the ratio of 2 to 4.

Fig. 47 shows drawings of two kinds of tool handles with their catalog numbers. The use of proportion may be illustrated by comparing the costs of the two kinds of handles. If No. 122 costs 12 cents and No. 123 costs 23 cents,

we may say that the cost of No. 122 is to the cost of No. 123 as 12 is to 23. By ratio and proportion this can be expressed as,

Cost of No. 122 : cost of No. 123 :: 12¢. : 23¢.

This can be written in the fractional form as follows:

$$\frac{\text{Cost of No. 122}}{\text{Cost of No. 123}} = \frac{12¢.}{23¢.}.$$

Two electricians had a wager as to the time it would take to do a job of wiring. Electrician *A* did the work in 27

FIG. 47. Tool Handles.

minutes, while it took electrician *B* $25\frac{1}{2}$ minutes. In the form of ratio and proportion this could be stated:

Speed of *A* : Speed of *B* :: 27 : $25\frac{1}{2}$

The quantities in a proportion are called *terms;* and they are numbered from left to right as follows:

First		*Second*		*Third*		*Fourth*
9	:	18	::	2	:	4

The first and fourth terms are called *extremes;* the second and third terms are called *means* (in the sense of middle). Thus, in the proportion 10 : 12 :: 5 : 6, the numbers 10 and 6 are the *extremes* and 12 and 5 are the *means.*

Important Principles of Proportion. You will find your work in proportion simple if you memorize and learn to apply these three important principles:

1. The product of the extremes is equal to the product of the means.
2. The product of the extremes divided by either mean gives the other mean.
3. The product of the means divided by either extreme gives the other extreme.

Examples. In the proportion 10 : 12 :: 5 : 6, the product of the extremes, 10 and 6, is equal to the product of the means, 12 and 5; that is,

$$10 \times 6 = 12 \times 5 = 60.$$

The product of the extremes (10×6) divided by the first mean, 12, will give the second mean, as $\frac{10 \times 6}{12} = 5$.

In like manner the product of the extremes (10×6) divided by the second mean, 5, will give the first mean, as $\frac{10 \times 6}{5} = 12$.

The product of the means (12×5) divided by the first extreme, 10, will give the second extreme, as $\frac{12 \times 5}{10} = 6$.

In like manner the product of the means (12×5) divided by the second extreme will give the first extreme, as $\frac{12 \times 5}{6} = 10$.

In the following examples a *missing* or *unknown quantity* has to be found. In mathematics unknown quantities are usually indicated for the sake of convenience by one of the last three letters of the alphabet. In this and other chapters the usual practice will be followed and the missing or unknown term will be designated by x or y or z.

Examples. In the proportion 36 : 18 :: 12 : x. Find the value of the missing term, x.

According to rule 3, the missing extreme, x, $= \frac{18 \times 12}{36} = 6$.

In the proportion $9 : x :: 6 : 24$ (see rule 2) the missing term, $x, = \frac{9 \times 24}{6} = 36.$

Pulleys and gears as used in shops suggest many examples of interesting ratios. Such problems are readily solved by methods of ratio and proportion, but require a little explanation to be readily understood. Everyone who has had anything to do with machinery knows that by belting from a pulley of large diameter running at a low number of revolutions per minute to a pulley of small diameter, the small pulley will make a much larger number of revolutions per minute than the large pulley. Common illustrations of this principle are to be seen in an ordinary sewing machine or in a bench lathe.

The same is as true of gears as of pulleys. When one gear meshes into another as shown in Fig. 48, the smaller gear must make more revolutions per minute than the larger. Roughly speaking, the principle is that as the size or the diameter of the smaller gear is reduced the number of its revolutions per minute is increased. (r.p.m. is the abbreviation of number of revolutions per minute.)

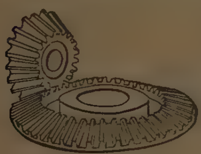


FIG. 48.
Bevel Gears.

The following illustrations (Figs. 49 and 50) show a typical example of the use of gears.

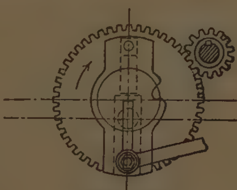


FIG. 49.
Special Gearing.

When two pulleys are connected by a belt every part of the belt is traveling at the same linear speed (as, for example, belt speed in feet per minute). Then, if the circumference (the distance around the rim) of the larger pulley A is twice the circumference of the smaller pulley B, pulley B must

make two turns or revolutions while pulley A makes one revolution. That is, B makes twice as many revolutions in a given time as A. In similar manner, if the circumference of pulley A is 3, 4, 5, 6, or 100 times larger than that of

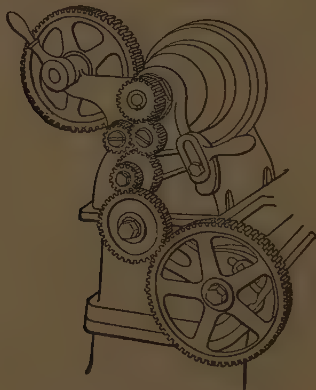


FIG. 50. Lathe Gears.

pulley B, pulley B will make 3, 4, 5, 6, or 100 times as many revolutions as pulley A. In like manner if the circumference of pulley A is $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{3}$, or $\frac{2}{5}$ of pulley B, pulley B will make $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{3}$, or $\frac{2}{5}$ as many revolutions as pulley A.

The following symbols will be used in all gear and pulley problems:

D = diameter of the large gear or pulley,
d = diameter of the small gear or pulley,
N = r.p.m. of large gear or pulley,
n = r.p.m. of small gear or pulley.

You should remember in arranging proportions that

The larger : smaller :: larger : smaller.

Bearing these principles in mind you will arrange your proportion as follows: $D : d :: n : N$.

You will have no difficulty in proportion if you determine correctly "the larger" and "the smaller" elements in all problems.

Example. If the diameter of a pulley is 36 inches and it makes 120 r p.m., what is the r.p.m. of a second pulley 16 inches in diameter belted to the first?

Solution. $D = 36$, $d = 16$, $N = 120$ r.p.m., $n =$ what?

$D : d :: n : N$, that is, $36 : 16 :: n : 120$.

$$n = \frac{36 \times 120}{16} = 270 \text{ r.p.m. of the smaller pulley.}$$

Check: Product of means = product of extremes; therefore, $120 \times 36 = 4320 = 270 \times 16 = 4320$.

Practical work in mathematics often requires some knowledge of three-sided figures called triangles. Fig. 51 illustrates triangles of different kinds.

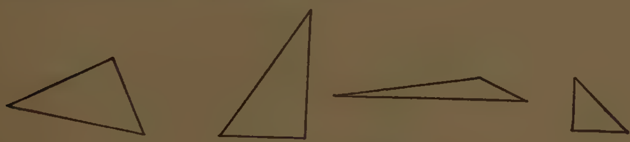


FIG. 51. Triangles.

Though triangles have a great variety of shapes they are alike in certain ways and it is possible to lay down rules about them which are always true. Fig. 52 illustrates a fact which is often used in practical calculations.

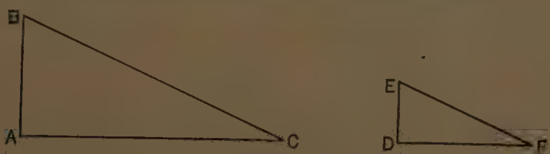


FIG. 52. Similar Right Triangles.

For instance, triangles *ABC* and *DEF* appear alike to the eye. But they are alike in a more exact sense. If you

measure accurately the angles of these triangles you will find that angle A equals (is as wide as) the corresponding angle D; angle B equals the corresponding angle E; and angle C equals the corresponding angle F.

You will also find by measurement that the length of the side AB has exactly the same relation to the corresponding side ED that the side BC has to EF, and AC to DF. In other words, the lengths of the corresponding sides form proportions as shown below. Triangles of which it may be said that the corresponding angles are equal and the corresponding sides are proportional are called *similar* triangles.

The most common form of triangle used is the right triangle, — a triangle in which one angle is a right angle. Triangles ABC and DEF in Fig. 52 are right triangles because angles A and D are right angles.

Now referring to the right triangles in Fig. 52, we can form the following proportions, using length of sides as terms, thus:

$$AB : ED = AC : DE$$
$$AC : DF = AB : DE$$
$$AB : AC = DE : DF, \text{ etc.}$$

A large number of combinations may be obtained from these. In going from one triangle to another the ratios used must be expressed in the *right order*. For example, the proportion cannot be written as $AB : AC :: DE : EF$ because the sides are not similar or are not in the same order on each side of the equality sign.

In practical problems it is customary to draw the two triangles together (Fig. 53). Thus, triangle ADE is similar to triangle ABC and we may form such proportions as

$$AD : AC :: AE : AB$$
$$AB : BC :: AE : DE$$
$$AE : AB :: DE : BC, \text{ etc.}$$

Example. This principle may be applied to a practical problem in finding the height of towers, chimneys and similar objects. In Fig. 54, H, the height of the tower is the distance to be determined.

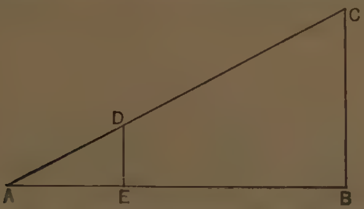


FIG. 53. Right Triangles Drawn Together.

Measure on the ground a level distance of 150 feet from the center of the base of the tower. From the point A, sight towards the top of the tower through the top of a stick set upright in the ground. Then measure the distance from A to E, (the bottom of

FIG. 54. Measuring a Tower.

the stick). We will find in this case that AE is twenty feet. Then measure the height of the upright stick ED, which we will find in this instance to be 10 feet.

Referring to Fig. 53 we find that our measuring has followed the lines of two similar right triangles of which certain dimensions are known.

For instance	the side AB equals 150 feet,
	the side EA equals 20 feet,
	the side ED equals 10 feet.

The corresponding angles are all equal (see Fig. 53). We now have the necessary terms to form a proportion and by this means can find H, the height of the tower.

Solution. $H : 150 :: 10 : 20.$

Referring to the three principles on page 68 we find that

$$20\,H = 10 \times 150, \quad \text{or} \quad H = \frac{10 \times 150}{20}.$$

By cancellation, $$H = \frac{\overset{5}{\cancel{10}} \times \overset{15}{\cancel{150}}}{\underset{10}{\cancel{20}}} = 75 \text{ feet.}$$

Example. Another interesting application of the comparison of different heights as determined by ratio and proportion may be found by comparing the heights of objects with their respective shadows.

Suppose, for example, that at a given time of the day a certain tree (Fig. 55) cast a shadow 52 feet long. How high is the tree?

Fig. 55. Measuring Height of Tree from Length of Shadows.

Is it evident from the preceding example that, *Height of tree* is to *height of stick* as *shadow of tree* is to *shadow of stick?*

This may be written $H : h :: S : s$. In this proportion, H is the quantity to be found, $h = 6$ feet, $S = 52$ feet and $s = 2\frac{1}{2}$ feet. Find H.

Solution. Substituting the known values in the proportion, we have

$$H : 6 :: 52 : 2.5.$$

That is, $$2.5\,H = 312 \text{ (see page 68).}$$

$$H = \frac{312}{2.5} = 124.8 \text{ feet (the height of the tree).}$$

It will be excellent practice for you actually to compute the height of some tall object, — a factory chimney, for example. You will be surprised at the accuracy of the result if your work is properly done.

PROBLEMS — GROUP I

1. It is found that a factory chimney casts a shadow of 115 feet at a time of day when an 8-foot stick casts a shadow of 5 feet. What is the height of the chimney?

2. Fig. 56 shows a treadle on a foot-power lathe. What is the distance of A. What is the ratio of A to B if B equals 9 inches? Express the last answer as a proportion.

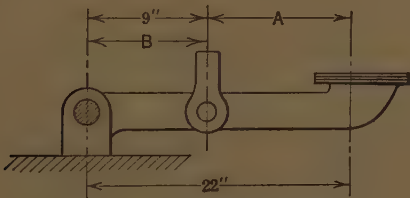


FIG. 56. Treadle of Lathe.

3. Find the weight of 3520 feet of copper wire if 1200 feet weigh 220 pounds. Indicate the proper proportion.

4. A man in an automobile travels 420 miles from 6 A.M. to 6 P.M. driving continuously. At the same rate how far does he travel in $20\frac{1}{2}$ hours?

5. The solution of grades requires a knowledge of the application of ratio as explained. For example, if a road rises $1\frac{1}{4}$ feet for every 100 feet of horizontal distance, the grade is said to be $1\frac{1}{4}$ per cent.

$$\frac{1\frac{1}{4}}{100} = .0125.$$

It is found that a certain section of State road rises 2 feet 9 inches in 400 feet of horizontal distance. What is the grade of this section?

6. If the grade in the preceding problem were uniform for the distance of two miles what would the total difference in elevation be between the beginning and end of the grade?

7. A miter box, Fig. 57 (of the dimensions shown), is made from a board 5 inches wide and 6 feet 6 inches long. Disregarding waste for saw cuts etc., what part of the original board is left?

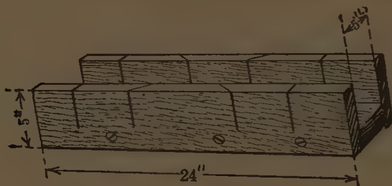


FIG. 57. Miter Box.

8. In making a rough estimate a builder remembers that a certain building containing 30,500 cubic feet cost $5800. What will a building of the same type cost which contains 48,000 cubic feet?

9. A certain casting weighs 320 pounds. If the ratio of the weight of the casting and the wooden pattern from which it was made is as 16 to 1, what is the weight of the pattern?

10. A countershaft pulley is $7\frac{5}{8}$ inches in diameter and the lathe pulley which it drives by a belt is $4\frac{5}{8}$ inches in diameter, what is the r.p.m. of the lathe spindle if the countershaft pulley makes 160 r.p.m.?

11. What diameter pulley should be put on a countershaft to run at 175 r.p.m. when belted to a pulley 15 inches in diameter on the main line shaft making 160 r.p.m.

12. Find the value of X in each of the following proportions:

$$(a)\ \tfrac{3}{5} : \tfrac{7}{8} :: X : \tfrac{5}{9}.$$
$$(b)\ \tfrac{9}{10} : \tfrac{21}{25} :: 3.5 : X.$$

13. There is a geometrical rule that the areas of two circles are to each other as the squares of their diameters. What is the ratio of the areas of the circles A and B? The diameter of A is 10 inches and B 4 inches. $A : B :: ? : ?$ (The square of a number is obtained by multiplying the number by itself. The square of 4 is 16.)

14. If two copper wires have diameters of .12 inches and .2 inches what is the ratio of their areas?

15. A spindle is driven by a train of gears as shown in Fig. 58, the first driver on the shaft has 50 teeth, the two stud driving gears each 50 teeth, and the followers each 125 teeth with 150 teeth in the gear on the spindle. What is the number of revolutions of the spindle for one turn on the shaft?

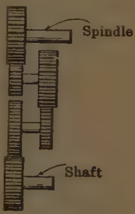


FIG. 58.
Train of Gears.

16. If it takes 82 pounds of metal to make 12 castings how much does it take to make 12 castings?

17. If a metal planer has a cutting speed of 35 feet per minute and the ratio of the cutting speed to the return speed is as 1 : 27, what is the return speed in feet per minute? (Return speed is the speed at which the planer slides back to position for a new cut.)

PROBLEMS — GROUP II

1. The diameter of a rubber covered wire is .427 inch including insulation. The diameter of the bare wire is .064 inch. What is the ratio of the area of the insulated wire to the area of the bare wire? (The ratio of the areas will be the same as the ratio of the diameters.)

2. Electric wires are often placed in pipes which protect them. The pipes are called conduits. Two wires having a total cross-sectional area of .1046 square inch are placed in a conduit having an area of .2523 square inch. What is the ratio of the wire area to the conduit area?

3. A reel of copper wire weighs 183.9 pounds. The insulation on the wire weighs 30.7 pounds. What is the ratio of the total weight to the weight of the insulation?

4. If the spacing between wires must be proportional to the voltage and the wires of a 300-volt circuit must be separated $2\frac{1}{2}$ inches, what must be the spacing of the wires of a 2400-volt circuit?

5. An electrician receives \$3.60 per day and his helper gets \$2.00.

If they both work the same number of days, how much will the helper receive for the same time that the electrician receives $67.50?

6. The weights of two generators are proportional to their capacities. If one generator is rated at 150 kilowatts and weighs 14,040 pounds, how much does the other, rated at 200 kilowatts, weigh?

7. If a 7-pound electric flat iron becomes hot in 8.68 minutes after being connected to the circuit, how many minutes will be required for a 12-pound iron, if the time necessary is proportional to the weight?

8. The weight of 10 feet of No. 1 insulated cable is approximately 4.2 pounds. What is the weight of 480 feet of the same cable? Work out as a problem in proportion.

9. The ratio of bare wire to the wire when insulated is 29 : 41 for a certain grade of wire. If the bare wire weighs 87 pounds, what is the weight of the same wire when insulated?

10. A flag pole casts a shadow 17 feet 3 inches in length when a telephone pole 30 feet high casts a shadow 8 feet $7\frac{1}{2}$ inches long. How high is the flag pole?

11. The length of the shadow of a telephone pole to the first cross arm is 22 feet. An 8-foot pole at the same time casts a shadow 3 feet long. What is the distance from the ground to the first cross arm?

CHAPTER VI

POWERS OF NUMBERS

This subject, **Powers of Numbers**, is important for the solution of many practical formulas that you will have to use. Study the following explanations until you are sure you understand their meaning.

A **Power** of a number is obtained by multiplying a number by itself one or more times; thus we call $2 \times 2 \times 2 \times 2 \times 2$ (= 32) the fifth power of 2.

The **Exponent of a Power** is a figure placed at the right and a little above the number to show the power to which the number is to be raised; thus, in 2^2, the exponent ^{2} means the second power of 2. The following are examples:

$$2 \times 2 = 2^2 = 4.$$

Here we say the 2 has been "*squared.*"

$$2 \times 2 \times 2 = 2^3 = 8.$$

Here we say the 2 has been "cubed."

$$2 \times 2 \times 2 \times 2 = 2^4 = 16.$$

Here we say the 2 has been raised to the fourth power.

When we write 2^5, we mean that 2 has been multiplied 5 times. We say that 2 has been raised to the fifth power. The same method of expressing powers applies to all other powers.

2^6 is read "2 to the sixth power."
2^7 is read "2 to the seventh power."

Interesting facts which should be remembered are the following:

1 raised to any power whatever is 1.

$1^1 = 1$ raised to the first power = 1.
$1^2 = 1$ " " " second " = 1.
$1^{27} = 1$ " " " twenty-seventh power = 1.

1 multiplied by itself any number of times always equals 1.

You have just learned (page 79) how to raise a *whole number* to a power. The raising of fractions to a power is quite simple: you merely multiply the fraction by itself the required number of times, that is, you raise both numerator and denominator to the required powers, thus

$$\left(\frac{2}{3}\right)^2 = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}.$$

$$\left(\frac{2}{3}\right)^3 = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{8}{27}.$$

$$\left(\frac{2}{3}\right)^4 = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \frac{16}{81}.$$

To raise a decimal to a power multiply it by itself exactly as if it were a whole number then place the decimal point as in the multiplication of decimals (see page 39).

$(.25)^2 = .25 \times .25 = .0625.$
$(2.62)^3 = 2.62 \times 2.62 \times 2.62 = 17.9847+.$

There is one application of this process which you should understand because of its value in solving formulas. Let the word formula have no terrors for you. A formula is really a time saver, the friend of busy men. A formula is a short-cut method of stating a mathematical truth in which symbols are used for words. It isn't necessary to memorize a long list of formulas; merely remember where they are to be found.

Often you see in formulas a quantity containing a letter which is raised to a power. This letter represents some number and if you do not understand what is to be done with it, difficulties are likely to arise.

Study the following carefully:

If A is squared the result is A^2. (1)

If $2A$ is squared the result is $2A \times 2A = 4A^2$. (2)

If $3A$ is cubed the result is $3A \times 3A \times 3A = 27A^3$. (3)

Suppose A (Fig. 59) represents the area of a square, then $2A$ would represent the area of a square twice as large, or, the areas of the two such squares *added together*. That is, $2A$ contains twice as many square inches or square feet (whatever the unit of measurement may be) as A.

If A contained 10 square inches a square containing 100 square inches could be represented by A^2, which is 10 times as large in area as A.

Note the difference between a square containing 20 square inches ($2A$) and a square containing 100 square inches (A^2). This illustrates the difference between two times a quantity and the square of a quantity.

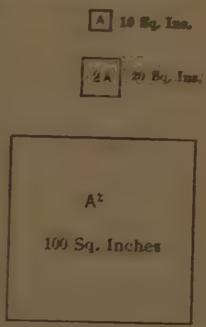


FIG. 59. Comparison of Areas.

PROBLEMS — GROUPS I AND II

1. What is the *second power* of 7? Note the difference between 2×7 and 7×7 (Fig. 60). 2×7 means that 7 is multiplied by 2. It is the same as $7 + 7$. 7^2 (the second power of seven) indicates 7×7.

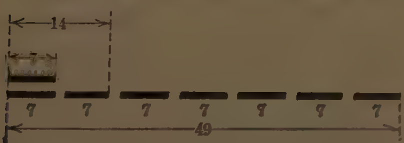


FIG. 60. Example of Squaring a Number.

2. What is the third power of 5?

3. What is the fourth power of 3?

4. How much is $(\frac{3}{7})^2$?

5. How much is $(1)^{1,000,000}$ — that is, 1 raised to the millionth power?

6. How much is $(.5)^3$?

7. What is the square of $2\frac{1}{3}$? (Change to a fraction before squaring.)

8. What is the fifth power of $\frac{1}{2}$?

9. What is the square of $3A$?

10. What is the eighth power of $\frac{1}{2}$?

11. Find the value of:

(a) 45^2. (b) 105^3. (c) 16^5.

12. Find the value of:

(a) $(\frac{14}{15})^2$. (b) $(13\frac{2}{3})^3$.

13. Find the value of:

(a) $(2.5)^2$. (b) $(2^4)^2$.

Note. — In (b) it is first necessary to raise 2 to the fourth power and then square the result.

Fig. 61. Lifting Jack.

14. Find the value of: $(\frac{4}{5})^3 \times (\frac{4}{5})^2$.

15. Find the value of: $(\frac{2}{3})^3 \times (\frac{3}{4})^4$. (Use cancellation here.)

16. Find the value of: $3^2 \times 7^3 \div (7^2 \times 3)$.

17. If a man using a lifting jack and exerting a force of 50 pounds at the end of the handle can lift a weight equal to $\frac{4}{5}$ of the square of the force he exerts, how much can he lift?

18. The force (pressure) of the wind in pounds per square inch may be determined by using the formula $P = .005\,V^2$, in which P equals the force and V equals the velocity in miles per hour. Find the force of the wind when blowing at the rate of 50 miles per hour.

CHAPTER VII

ROOTS OF NUMBERS

The root of a number is just the reverse of its power. For example, $2 \times 2 = 4$, which is the square, or second power, of 2. The number 2 is, therefore, a root (square root) of 4. $4 \times 4 \times 4 \times 4 = 264$, which is the fourth power of 4. So 4 is the fourth root of 264. $3 \times 3 \times 3 = 27$, which is the third power (cube) of 3. Then 3 is the third root (cube root) of 27. The above should show you that any number may be a root of *some number*.

The same principles apply to fractions. If $\frac{2}{3}$ multiplied by itself produces $\frac{4}{9}$, $\frac{2}{3}$ is the square root of $\frac{4}{9}$. Note in this case that the square root of $\frac{4}{9}$ may be obtained by taking the square root, first of the numerator and then of the denominator and putting the results down in the same order as in the original fraction. The same is true of all fractions having for numerators and denominators numbers which are the product of some number multiplied by itself. In the same manner,

$\frac{1}{2}$ is the square root of $\frac{1}{4}$,
$\frac{3}{4}$ is the cube root of $\frac{27}{64}$,
$\frac{2}{3}$ is the fifth root of $\frac{32}{243}$.

You will remember that 1 raised to any power is 1. Any root, therefore, of 1 is 1, thus

The square root of 1 is 1.
The cube root of 1 is 1.
The fourth root of 1 is 1, etc.

Now it is very convenient to indicate what root of a number is to be taken by using the root sign, which is usually written before the number of which a root is to be found.

The root * sign is $\sqrt{\ }$.

When there is no number written over the root sign, it means the square root of the number covered by the sign, thus $\sqrt{4}$ means the square root of 4, $\sqrt[3]{8}$ means the cube (third) root of 8, etc.

$\sqrt{16}$	denotes	the	square root of 16,
$\sqrt[3]{64}$	"	"	cube root of 64,
$\sqrt[5]{32}$	"	"	fifth root of 32,
$\sqrt[3]{A^3}$	"	"	cube root of A^3, that is, A.

These are all simple processes, and you should be able to solve them mentally. It is necessary, however, that you be able to extract roots of larger quantities. This refers especially to numerical quantities and to the extraction of the square root. Roots greater than the square root are not frequently used and their solution may be omitted in practical work.

Study carefully the following solutions of square root until you understand them thoroughly.

Finding the Square Root. The first thing to do when you wish to find the square root of a whole number is *to begin at the right* and separate or point off the number into groups of two figures each, as follows:

23′56′94′.

The same holds true for any *whole* number, that is, a number containing no decimal.

The number 15621 is pointed off 1′56′21.

* This is also sometimes called a radical sign.

These two numbers (235694 and 15621) are given because one contains an even number of figures, and the other an odd number of figures.

To illustrate how the square root is taken let us find the square root of 97344. The following rules should be observed:

1st. *Separate the given number into periods, or groups, of two figures each* beginning at the right and pointing towards the left. We then have 9′73′44. If the given number contains an odd number of figures the last period at the left will contain only *one* figure.

2nd. *Find the greatest number whose square is the same or less than the first left-hand period, or group, and place it as the first figure in the quotient.*

The greatest number in this case is 3. 3 is the square root of 9 and will be the first figure in the quotient.

	9′73′44	312
	9	
61	73	
	61	
622	1244	
	1244	

3rd. *Square the number obtained in the quotient and place the result under the first left-hand period. Subtract, and add the next period if there is a remainder.*

In this case there will be no remainder because the first number of which the square root is taken is a perfect square. Add the next period, 73.

4th. *To obtain the trial divisor multiply the number already found in the quotient by* 2 ($2 \times 3 = 6$ in this case) *and bring it down as the first figure in the trial divisor. Ascertain how many times this number will go into the dividend* (in this case 73) *exclusive of the right-hand figure* (3).

This means that 7 is to be divided by 6 to obtain the next figure in the quotient and also the next figure in the trial divisor, which

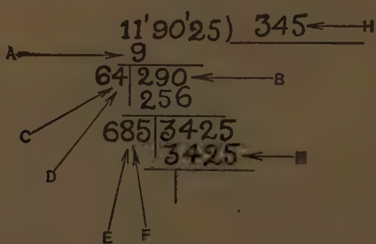
is 1 in this case. So we place 1 next to 6 making the trial divisor 61. The number 61 divides into 73 once so we place 1 adjacent to 3 in the quotient and get 31. Multiplying 61 by 1, placing the product under 73 and subtracting we have 12 as the remainder. We then proceed as before. That is, we set down the next period (44) to 12, multiply 31 by 2 for the next trial divisor, etc. and get 62 for the first part of the trial divisor.

As the number 62 will divide into 124 twice we write 2 as the next figure in the trial divisor and also in the quotient and 2 times 622 equals 1244. This is exactly the same as the remainder previously obtained. The root is now complete because no further operation is possible.

The above calculation shows that the square root of 97344 is 312.

Other illustrations of square root follow. Study them carefully and as you study them be sure that you understand the application of the above rules.

The next illustration differs from the one previously given in that the first period is not a *perfect square*. The problem is to take the square root of 119025.



The key to the above work is as follows:

A. 9 is the nearest square less than 11. The square root of 9 is 3, the first figure in the quotient.

B. Here we subtract 9 from 11 and add the next period which is 90.

C. 6 is obtained by multiplying 3 (first figure in the quotient) by 2.

D. As 6 divides in 29 four (4) times, 4 becomes the second figure in the quotient.

E. 68 is obtained by multiplying 34 by 2.

F. 5 is obtained by dividing 342 by 68, and becomes the next figure in the quotient.

G. 3425 is obtained by multiplying 685 by 5.

H. As there is no remainder after the above operations are completed 345 is the exact square root of 119025.

Other illustrations are as follows:

Find the square root of 5273.5267.

```
        52′73′.52′67   )72.61+
        49
  142 | 373
      | 284
 1446 | 8952
      | 8676
14521 |  27667
      |  14521
```

Note here that beginning at the decimal point the periods are formed by grouping from the decimal point towards the right. The plus sign shows that the calculation may be carried further if necessary.

Find the square root of .00009216

```
  .00′00′92′16   ).0096
   00
     00
     00
       92
       81
186 | 1116
      1116
```

This problem should be helpful to you because of the way ciphers are handled. Note that they are pointed off in pairs just as other figures and that in the root (the result) *one* cipher represents the root. Note also that pointing off groups, or periods, is *towards the right from the decimal point.*

It is possible to take the fourth root of a number by finding the square root of the square root.

Thus the square root of 16 is 4, and the square root of 4 is 2.

$$\sqrt[4]{16} = \sqrt{\sqrt{16}} = \sqrt{4} = 2.$$

Therefore, 2 is the fourth root of 16.

This simple solution is well worth your consideration because exactly the same method would be applied to larger numbers.

For example, to find the fourth root of 277729 we would first take the square root of it (527) and then the square root of 527 (22.95). That is, 22.95 is almost exactly the fourth root of 277729. It is the number which if multiplied four times would produce 277729. The root really falls between 22.95 and 22.96.

The principal reason for your needing to know how to find square root is because of its general use in formulas and in right angle triangle problems. The latter will be treated in a later chapter but it will do no harm to begin to think about it here.

In the right triangle (Fig. 62), a definite relation exists between the two perpendicular sides (AB and BC) and the longer side AC.

FIG. 62. Right Triangle.

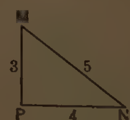


FIG. 63.
Right Triangle Problem.

The square of AC equals the sum of the squares of AB and BC. This may be expressed by the equation $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$. The square of either of the perpendicular sides equals the square of the longer side minus the square of the other perpendicular side.

That is, $\overline{AB}^2 = \overline{AC}^2 - \overline{BC}^2$ and $\overline{BC}^2 = \overline{AC}^2 - \overline{AB}^2$.

In Fig. 63, if $MN = 5$, $NP = 4$,

and $MP = 3$, $\overline{MN}^2 = \overline{MP}^2 - \overline{NP}^2$.

That is, $25 = 9 + 16$.

In a square the area equals the square of the side. If the side of a square (Fig. 64) is 3 inches, its area is equal to $(3)^2$ or 9 square inches.

Suppose that one wished to find the length of the stringer for the stairs in Fig. 65.

$$\begin{aligned}(\text{Length of stringer})^2 &= (9.5)^2 + (12)^2\\ &= 90.25 + 144,\\ &= 234.25.\\ \text{Length} &= \sqrt{234.25}\\ &= 15.3 \text{ feet.}\end{aligned}$$

Another interesting application of this rule depends on the fact that a right angle may always be constructed by taking the sides of the triangle in the relation 3 to 4 to 5. This relation is very easily remembered because the numbers follow exactly in order, 3, 4, 5. How the rule is applied is shown below.

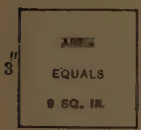


Fig. 64. Square.

Drive a nail in a board at A (Fig. 66). Measure off in any direction the line AB equal to 4 inches and drive nail B. Tie a string to nail A and, measuring off 3 inches on the string, attach a pencil. Now you have a home made compass. Stretching the string tight draw with the pencil on the board a section of a circle, or arc. Then measuring off 5 inches on the string attach the pencil at that point. Tie the string to nail B and draw on the board another arc so that it will cut across the first arc at C.

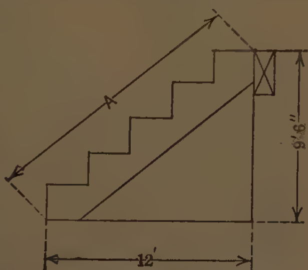


Fig. 65. Stringer for Stairway.

Straight lines drawn through points A, B, C, will form a right triangle, the angle BAC being a right angle.

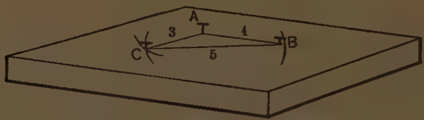


FIG. 66. Laying-out Board.

If a larger area was to be laid off, on the ground for example, numbers twice or three times as great as 3, 4 and 5 may be taken, such as

6, 8 and 10
or 9, 12 and 15, etc.

The use of the square root in a formula is well illustrated in the following solution.

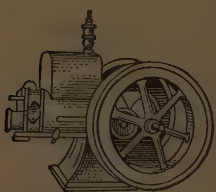


FIG. 67. Gasoline Engine.

The S. A. E. (Society of Automobile Engineers) standard formula for a gasoline engine (Fig. 67) horse power is $\frac{D^2 \times N}{2.5}$ based upon 1000 feet per minute piston speed.

In this formula D is the cylinder diameter in inches, N the number of cylinders, and 2.5 a constant based on the general opinion of experts as to a conservative rating.

Suppose the diameter of the cylinder (D) is to be found when $N = 4$ and $h.p.$ (the horse power) $= 45$. In other words, solve for D when $N = 4$ and $h.p. = 45$. This will require finding a square root.

The formula to be used is $h.p. = \frac{D^2 \times N}{2.5}$.

Substituting the known values in this equation we have

$$45 = \frac{D^2 \times 4}{2.5}.$$

$D^2 \times 4$ is the same as $4\,D^2$, so the equation may be written

$$45 = \frac{4\,D^2}{2.5}.$$

It is customary in an equation to have the unknown quantity (the quantity D in this case) on the left side of the equality sign. The above will therefore be written as

$\frac{4\,D^2}{2.5} = 45$, which may be reduced, since 4 divided by 2.5 is 1.6.

Thus, $4\,D^2 \div 2.5 = 1.6\,D^2.$

We can then say

$$1.6\,D^2 = 45.$$

From this $D^2 = \frac{45}{1.6}$ or $\frac{45}{1.6} = 28.12.$

If $D^2 = 28.12$, D = the square root of 28.12. This may be written $D = \sqrt{28.12}$. Taking the square root of 28.12, we have $D = 5.3$. The diameter of the cylinder will be 5.3 inches.

Problems — Group I

1. Find the square root of the following:
(a) 6561. (b) 11664.

2. Find the square root of the following:
(a) .0841. (b) .001225.

3. Find the square root of 89526.025681.

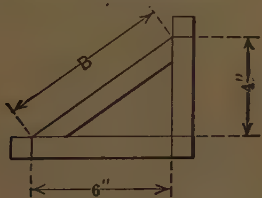


Fig. 68. Wooden Brace.

4. What is the value of $\sqrt{7}$. (Carry the result to four decimal places.)

5. If $6\,A^2 = 864$, what is the value of A?

6. What is the length of the brace B in Fig. 68.

7. What is the distance Y from center to center of the pulley in Fig. 69?

8. A square nut (Fig. 70) is $3\frac{1}{8}$ inches on a side as shown. What is the distance "across corners"?

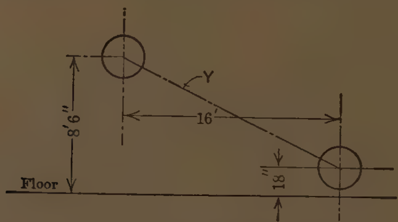


FIG. 69. Pulley Problem.

9. If the distance from corner to corner of a square room is 98 feet, what is the distance along each side of the room?

10. What length of wire must be used to make a guy rope as shown in Fig. 71 if we allow 7 feet for fastening at the ends?

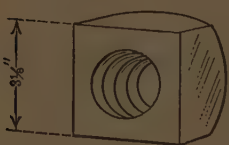


FIG. 70. Square Nut.

11. A ladder (Fig. 72) 20 feet long is set with its lower end 8 feet from the base of a vertical wall and with its top resting against the wall. What is the distance from the top of the ladder to the bottom of the wall?

12. Two fence posts each 4 feet high are 8 feet apart. How long should a wire brace be made to reach from the bottom of one to a point 6 inches below the top of the other?

PROBLEMS — GROUP II

1. What is the difference in the number of square feet of slate required for two switchboards if one has 12 panels 30 inches wide and 5 feet high, and the other has 8 panels $42\frac{3}{4}$ inches wide and 4 feet high? If the electrical equipment for these boards averages 2.4 pounds per square foot of panel surface, how many pounds does the equipment for the 12-panel board weigh?

2. It was decided that a substation 186 feet long by 98 feet wide should have its floor space increased by 1302 square feet.

How much must be added to the length if the width is to be increased 7 feet?

3. An electric oven must have a base of 242.2 square inches. If it is 14 inches wide, how deep is it?

4. A machine requires insulating fiber as follows: 6 pieces 6 inches square and 12 pieces 3 inches square. If these pieces are

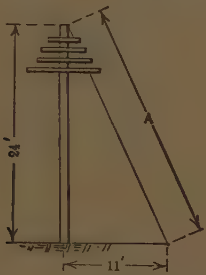


FIG. 71. Guy Rope Problem.

all cut from a large piece of fiber, how many square inches must it contain?

5. Find the area of a square marble switchboard panel 19 inches on a side.

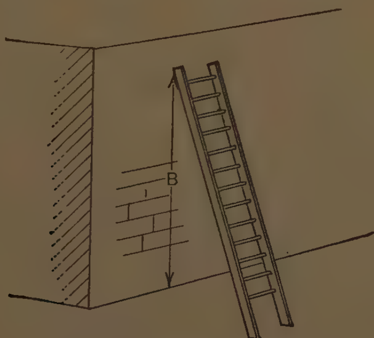


FIG. 72. Ladder Problems.

6. A factory consists of a room 47 feet on each side, a room 65 feet square. and an addition containing 127 square feet. How

many square feet of floor space are there? If there are 25 machines in the factory, what is the average number of square feet of floor space per machine?

7. The cool air enters a transformer room through a square pipe which is attached to and has the same area as a rectangular pipe 5 feet 4 inches wide and 16 inches high.

(*a*) What is the size of the square pipe?

(*b*) Which would be the cheaper to build, one foot of length of the square pipe or the same length of the rectangular pipe? Assume the same price per square foot.

TABLE I

Fraction	Dec. equiv.	Square	Square root	Cube	Cube root	Circum. circle	Area circle
$\frac{1}{64}$	.01563	.000244	.1250	.000003815	.2500	.04909	.000192
$\frac{1}{32}$	.03125	.000976	.1768	.00003052	.3150	.09818	.000767
$\frac{1}{16}$	.0625	.003906	.2500	.0002442	.3968	.1963	.003068
$\frac{3}{32}$	.09375	.008789	.3062	.0008240	.4543	.2945	.006903
$\frac{1}{8}$	.12500	.015625	.3535	.001953	.5000	.3927	.01228
$\frac{5}{32}$	.15625	.02441	.3953	.003815	.5386	.4909	.01916
$\frac{3}{16}$	.18750	.03516	.4330	.006592	.5724	.5890	.02761
$\frac{7}{32}$	.21875	.04786	.4677	01047	.6025	.6872	.03758
$\frac{1}{4}$	.25000	.06250	.5000	.01562	.6300	.7854	.04909
$\frac{9}{32}$	.28125	.07910	.5303	.02225	.6552	.8836	.06213
$\frac{5}{16}$	.3125	.09766	.5590	.03052	.6786	.9817	.07670
$\frac{11}{32}$	.34375	.11829	.5863	04062	.7005	1.080	.09281
$\frac{3}{8}$	.37500	.14063	.6124	05273	.7211	1.178	.1104
$\frac{13}{32}$	.40625	.16504	.6374	.06705	.7406	1.276	.1296
$\frac{7}{16}$	.43750	.19141	.6614	.08374	.7592	1.374	.1503
$\frac{15}{32}$	.46875	.2197	.6847	1030	.7768	1.473	.1726
$\frac{1}{2}$	.50000	.2500	.7071	.1250	.7937	1.571	.1963
$\frac{17}{32}$	.53125	.2822	.7289	.1499	.8099	1.669	.2217
$\frac{9}{16}$	.5625	.3164	.7500	.1780	8255	1.767	.2485
$\frac{19}{32}$	.59375	.3525	.7706	2093	.8405	1.865	.2769
$\frac{5}{8}$	.6250	.3906	.7906	2441	.8550	1.963	.3068
$\frac{21}{32}$	.65625	.4307	.8101	2826	.8690	2.062	.3382
$\frac{11}{16}$	.6875	.4727	.8292	3250	.8826	2.160	.3712
$\frac{23}{32}$	.71875	.5166	.8478	3713	.8958	2.258	.4057
$\frac{3}{4}$	.7500	.5625	.8660	4219	.9086	2.356	.4418
$\frac{25}{32}$	.78125	.6104	.8839	4768	.9210	2.454	.4794
$\frac{13}{16}$	.8125	.6602	.9014	5364	.9331	2.553	.5185
$\frac{27}{32}$	.84375	.7119	.9186	6007	.9449	2.651	.5592
$\frac{7}{8}$	.8750	.7656	.9354	.6699	.9565	2.749	.6013
$\frac{29}{32}$	.90625	.8213	.9520	.7443	.9677	2.847	.6450
$\frac{15}{16}$	.9375	.8789	.9682	8240	.9787	2.945	.6903
$\frac{31}{32}$	.96875	.9385	.9843	9091	.9895	3.043	.7371
1	1	1	1	1	1	3.1416	.7854

TABLE II

No.	Square	Cube	Square root	Cube root	Diam-eter	Area	Circum-ference
1	1	1	1.0000	1.0000	1	.785	3.141
2	4	8	1.4142	1.2599	2	3.141	6.283
3	9	27	1.7321	1.4422	3	7.068	9.424
4	16	64	2.0000	1.5874	4	12.56	12.566
5	25	125	2.2361	1.7100	5	19.63	15.708
6	36	216	2.4495	1.8171	6	28.27	18.849
7	49	343	2.6458	1.9129	7	38.48	21.991
8	64	512	2.8284	2.0000	8	50.26	25.132
9	81	729	3.0000	2.0801	9	63.61	28.274
10	100	1,000	3.1623	2.1544	10	78.54	31.416
11	121	1,331	3.3166	2.2240	11	95.03	34.557
12	144	1,728	3.4641	2.2894	12	113.09	37.699
13	169	2,197	3.6056	2.3513	13	132.73	40.84
14	196	2,744	3.7417	2.4101	14	153.93	43.982
15	225	3,375	3.8730	2.4662	15	176.71	47.124
16	256	4,096	4.0000	2.5198	16	201.06	50.265
17	289	4,913	4.1231	2.5713	17	226.98	53.407
18	324	5,832	4.2426	2.6207	18	254.47	56.548
19	361	6,859	4.3589	2.6684	19	283.52	59.69
20	400	8,000	4.4721	2.7144	20	314.16	62.832
21	441	9,261	4.5826	2.7589	21	346.36	65.973
22	484	10,648	4.6904	2.8020	22	380.13	69.115
23	529	12,167	4.7958	2.8439	23	415.47	72.256
24	576	13,824	4.8990	2.8845	24	452.39	75.398
25	625	15,625	5.0000	2.9240	25	490.87	78.54
26	676	17,576	5.0990	2.9625	26	530.93	81.681
27	729	19,683	5.1962	3.0000	27	572.55	84.823
28	784	21,952	5.2915	3.0366	28	615.75	87.964
29	841	24,389	5.3852	3.0723	29	660.52	91.106
30	900	27,000	5.4772	3.1072	30	706.86	94.248
31	961	29,791	5.5678	3.1414	31	754.79	97.389
32	1024	32,768	5.6569	3.1748	32	804.25	100.53
33	1089	35,937	5.7446	3.2075	33	855.35	103.67
34	1156	39,304	5.8310	3.2396	34	907.92	106.81
35	1225	42,875	5.9161	3.2711	35	962.11	109.95
36	1296	46,656	6.0000	3.3019	36	1017.8	113.09
37	1369	50,653	6.0828	3.3322	37	1075.2	116.23
38	1444	54,872	6.1644	3.3620	38	1134.1	119.38
39	1521	59,319	6.2450	3.3912	39	1194.5	122.52
40	1600	64,000	6.3246	3.4200	40	1256.6	125.66

TABLE II — *Continued*

No.	Square	Cube	Square root	Cube root	Diameter	Area	Circumference
41	1681	68,921	6.4031	3.4482	41	1320.2	128.80
42	1764	74,088	6.4807	3.4760	42	1385.4	131.94
43	1849	79,507	6.5574	3.5034	43	1452.2	135.08
44	1936	85,184	6.6332	3.5303	44	1520.5	138.23
45	2025	91,125	6.7082	3.5569	45	1590.4	141.37
46	2116	97,336	6.7823	3.5830	46	1661.0	144.51
47	2209	103,823	6.8557	3.6088	47	1734.9	147.65
48	2304	110,592	6.9282	3.6342	48	1809.5	150.79
49	2401	117,649	7.0000	3.6593	49	1885.7	153.93
50	2500	125,000	7.0711	3.6840	50	1963	157.1
51	2601	132,651	7.1414	3.7084	51	2042	160.2
52	2704	140,608	7.2111	3.7325	52	2123	163.4
53	2809	148,877	7.2801	3.7563	53	2206	166.6
54	2916	157,464	7.3485	3.7798	54	2290	169.6
55	3025	166,375	7.4162	3.8030	55	2375	172.8
56	3136	175,616	7.4833	3.8259	56	2463	175.9
57	3249	185,193	7.5498	3.8485	57	2551	179.1
58	3364	195,112	7.6158	3.8709	58	2642	182.2
59	3481	205,379	7.6811	3.8930	59	2733	185.4
60	3600	216,000	7.7460	3.9149	60	2817	188.5
61	3721	226,981	7.8102	3.9365	61	2922	191.6
62	3844	238,328	7.8740	3.9579	62	3019	194.8
63	3969	250,047	7.9373	3.9791	63	3117	197.9
64	4096	262,144	8.0000	4.0000	64	3217	201.1
65	4225	274,625	8.0623	4.0207	65	3318	204.2
66	4356	287,496	8.1240	4.0412	66	3421	207.3
67	4489	300,763	8.1854	4.0615	67	3525	210.5
68	4624	314,432	8.2462	4.0817	68	3631	213.6
69	4761	328,509	8.3066	4.1016	69	3739	216.8
70	4900	343,000	8.3666	4.1213	70	3848	219.9
71	5041	357,911	8.4261	4.1408	71	3959	223.1
72	5184	373,248	8.4853	4.1602	72	4071	226.2
73	5329	389,017	8.5440	4.1793	73	4185	229.3
74	5476	405,224	8.6023	4.1983	74	4300	232.9
75	5625	421,875	8.6603	4.2172	75	4417	235.6
76	5776	438,976	8.7178	4.2358	76	4536	238.8
77	5929	456,533	8.7750	4.2543	77	4656	241.9
78	6084	474,552	8.8318	4.2727	78	4778	245.0
79	6241	493,039	8.8882	4.2908	79	4901	248.2
80	6400	512,000	8.9443	4.3089	80	5026	251.3

TABLE II — *Continued*

No.	Square	Cube	Square root	Cube root	Diameter	Area	Circumference
81	6,561	531,441	9.0000	4.3267	81	5,153	254.5
82	6,724	551,368	9.0554	4.3445	82	5,281	257.6
83	6,889	571,787	9.1104	4.3621	83	5,410	260.8
84	7,056	592,704	9.1652	4.3795	84	5,541	263.9
85	7,225	614,125	9.2195	4.3968	85	5,674	267.0
86	7,396	636,056	9.2736	4.4140	86	5,808	270.2
87	7,569	658,503	9.3276	4.4310	87	5,944	273.3
88	7,744	681,472	9.3808	4.4480	88	6,082	276.5
89	7,921	704,969	9.4340	4.4647	89	6,221	279.6
90	8,100	729,000	9.4868	4.4814	90	6,361	282.8
91	8,281	753,571	9.5394	4.4979	91	6,503	285.9
92	8,464	778,688	9.5917	4.5144	92	6,647	289.0
93	8,649	804,357	9.6437	4.5307	93	6,792	292.2
94	8,836	830,584	9.6954	4.5468	94	6,939	295.3
95	9,025	857,375	9.7468	4.5629	95	7,088	298.5
96	9,216	884,736	9.7980	4.5789	96	7,238	301.6
97	9,409	912,673	9.8489	4.5947	97	7,389	304.7
98	9,604	941,192	9.8995	4.6104	98	7,542	307.9
99	9,801	970,299	9.9499	4.6261	99	7,697	311.0
100	10,000	1,000,000	10.0000	4.6416	100	7,854	314.1
101	10,201	1,030,301	10.0499	4.6570	101	8,012	317.3
102	10,404	1,061,208	10.0995	4.6723	102	8,171	320.4
103	10,609	1,092,727	10.1489	4.6875	103	8,332	323.5
104	10,816	1,124,864	10.1980	4.7027	104	8,494	326.7
105	11,025	1,157,625	10.2470	4.7177	105	8,659	329.8
106	11,236	1,191,016	10.2956	4.7326	106	8,824	333.0
107	11,449	1,225,043	10.3441	4.7475	107	8,992	336.1
108	11,664	1,259,712	10.3923	4.7622	108	9,160	339.2
109	11,881	1,295,029	10.4403	4.7769	109	9,331	342.4
110	12,100	1,331,000	10.4881	4.7914	110	9,503	345.5
111	12,321	1,367,631	10.5357	4.8059	111	9,676	348.7
112	12,544	1,404,928	10.5830	4.8203	112	9,852	351.8
113	12,769	1,442,897	10.6301	4.8346	113	10,028	355.0
114	12,996	1,481,544	10.6771	4.8488	114	10,027	358.1
115	13,225	1,520,875	10.7238	4.8629	115	10,386	361.1
116	13,456	1,560,896	10.7703	4.8770	116	10,568	364.4
117	13,689	1,601,613	10.8167	4.8910	117	10,751	367.5
118	13,924	1,643,032	10.8628	4.9049	118	10,935	370.7
119	14,161	1,685,159	10.9087	4.9187	119	11,220	373.8
120	14,400	1,728,000	10.9545	4.9324	120	11,309	376.9

TABLE II — *Continued*

No.	Square	Cube	Square root	Cube root	Diameter	Area	Circumference
121	14,641	1,771,561	11.0000	4.9461	121	11,499	380.1
122	14,884	1,815,848	11.0454	4.9597	122	11,689	383.2
123	15,129	1,860,867	11.0905	4.9732	123	11,882	386.4
124	15,376	1,906,624	11.1355	4.9866	124	12,076	389.5
125	15,625	1,953,125	11.1803	5.0000	125	12,277	392.7
126	15,876	2,000,376	11.2250	5.0133	126	12,469	395.8
127	16,129	2,048,383	11.2694	5.0265	127	12,667	398.9
128	16,384	2,097,152	11.3137	5.0397	128	12,867	402.1
129	16,641	2,146,689	11.3578	5.0528	129	13,069	405.2
130	16,900	2,197,000	11.4018	5.0658	130	13,273	408.4
131	17,161	2,248,091	11.4455	5.0788	131	13,478	411.5
132	17,424	2,299,968	11.4891	5.0916	132	13,684	414.6
133	17,689	2,352,637	11.5326	5.1045	133	13,892	417.8
134	17,956	2,406,104	11.5758	5.1172	134	14,102	420.9
135	18,225	2,460,375	11.6190	5.1299	135	14,313	424.1
136	18,496	2,515,456	11.6619	5.1426	136	14,526	427.2
137	18,769	2,571,353	11.7047	5.1551	137	14,741	430.3
138	19,044	2,628,072	11.7473	5.1676	138	14,957	433.5
139	19,321	2,685,619	11.7898	5.1801	139	15,174	436.6
140	19,600	2,744,000	11.8322	5.1925	140	15,393	439.8
141	19,881	2,803,221	11.8743	5.2048	141	15,614	442.9
142	20,164	2,863,288	11.9164	5.2171	142	15,886	446.1
143	20,449	2,924,207	11.9583	5.2293	143	16,060	449.2
144	20,736	2,985,984	12.0000	5.2415	144	16,286	452.3
145	21,025	3,048,625	12.0416	5.2536	145	16,530	455.4
146	21,316	3,112,136	12.0830	5.2656	146	16,741	458.6
147	21,609	3,176,523	12.1244	5.2776	147	16,971	461.8
148	21,904	3,241,792	12.1655	5.2896	148	17,203	464.9
149	22,201	3,307,949	12.2066	5.3015	149	17,436	468.0
150	22,500	3,375,000	12.2474	5.3133	150	17,671	471.2
151	22,801	3,442,951	12.2882	5.3251	151	17,907	474.3
152	23,104	3,511,808	12.3288	5.3368	152	18,145	477.5
153	23,409	3,581,577	12.3693	5.3485	153	18,385	480.6
154	23,716	3,652,264	12.4097	5.3601	154	18,626	483.8
155	24,025	3,723,875	12.4499	5.3717	155	18,869	486.9
156	24,336	3,796,416	12.4900	5.3832	156	19,113	490.0
157	24,649	3,869,893	12.5300	5.3947	157	19,359	493.2
158	24,964	3,944,312	12.5698	5.4061	158	19,607	496.3
159	25,281	4,019,679	12.6095	5.4175	159	19,855	499.5
160	25,600	4,096,000	12.6491	5.4288	160	20,106	502.6

TABLE II — *Concluded*

No.	Square	Cube	Square root	Cube root	Diameter	Area	Circumference
161	25,921	4,173,281	12.6886	5.4401	161	20,358	505.7
162	26,244	4,251,528	12.7279	5.4514	162	20,612	508.9
163	26,569	4,330,747	12.7671	5.4626	163	20,867	512.0
164	26,896	4,410,944	12.8062	5.4737	164	21,124	515.2
165	27,225	4,492,125	12.8452	5.4848	165	21,382	518.3
166	27,556	4,574,296	12.8841	5.4959	166	21,642	521.5
167	27,889	4,657,463	12.9228	5.5069	167	21,904	524.6
168	28,224	4,741,632	12.9615	5.5178	168	22,167	527.8
169	28,561	4,826,809	13.0000	5.5288	169	22,431	530.9
170	28,900	4,913,000	13.0384	5.5397	170	22,698	534.0
171	29,241	5,000,211	13.0767	5.5505	171	22,965	537.2
172	29,584	5,088,448	13.1149	5.5613	172	23,235	540.3
173	29,929	5,177,717	13.1529	5.5721	173	23,506	543.5
174	30,276	5,268,024	13.1909	5.5828	174	23,778	546.5
175	30,625	5,359,375	13.2288	5.5934	175	24,025	549.7
176	30,976	5,451,776	13.2665	5.6041	176	24,328	552.9
177	31,329	5,545,233	13.3041	5.6147	177	24,605	556.0
178	31,684	5,639,752	13.3417	5.6252	178	24,884	559.2
179	32,041	5,735,339	13.3791	5.6357	179	25,165	562.3
180	32,400	5,832,000	13.4164	5.6462	180	25,446	565.4
181	32,761	5,929,741	13.4536	5.6567	181	25,730	568.6
182	33,124	6,028,568	13.4907	5.6671	182	26,015	571.1
183	33,489	6,128,487	13.5277	5.6774	183	26,302	574.9
184	33,856	6,229,504	13.5647	5.6877	184	26,590	578.0
185	34,225	6,331,625	13.6015	5.6980	185	26,880	581.1
186	34,596	6,434,856	13.6382	5.7083	186	27,171	584.3
187	34,969	6,539,203	13.6748	5.7185	187	27,464	587.4
188	35,344	6,644,672	13.7113	5.7287	188	27,759	590.6
189	35,721	6,751,269	13.7477	5.7388	189	28,055	593.7
190	36,100	6,859,000	13.7840	5.7489	190	28,352	596.9
191	36,481	6,967,871	13.8203	5.7590	191	28,652	600.0
192	36,864	7,077,888	13.8564	5.7690	192	28,952	603.1
193	37,249	7,189,057	13.8924	5.7790	193	29,253	606.3
194	37,636	7,301,384	13.9284	5.7890	194	29,559	609.4
195	38,025	7,414,875	13.9642	5.7989	195	29,864	612.6
196	38,416	7,529,536	14.0000	5.8088	196	30,171	615.7
197	38,809	7,645,373	14.0357	5.8186	197	30,480	618.8
198	39,204	7,762,392	14.0712	5.8285	198	30,790	622.0
199	39,601	7,880,599	14.1067	5.8383	199	31,102	625.1
200	40,000	8,000,000	14.1421	5.8480	200	31,416	628.3

CHAPTER VIII

PROBLEMS ON INDUSTRIAL APPLIANCES AND POWER

The explanations in this chapter are about the methods of working out problems dealing with mechanical devices and electrical power, and solutions of many illustrative examples are given. When you understand these examples you should have very little trouble in solving almost any kind of similar problems. The principal subjects included are those of the applications of levers and the calculations of mechanical power.

Levers. The simplest form of a lever is a bar or rod supported on a pivot or fulcrum on which it is free to move. The pivot or fulcrum is usually marked in figures with F.

Levers are divided into three kinds or classes, according to the position of the pivot or fulcrum with respect to the position of the weight to be lifted and the power to be used.

A lever of the *first class* is shown in Figs. 73 and 74. Here the weight W is at one end, the force P is at the other end, and the pivot or fulcrum is between the weight and the force.

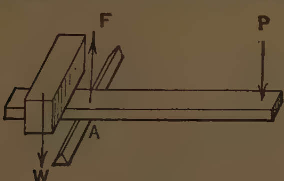


FIG. 73.

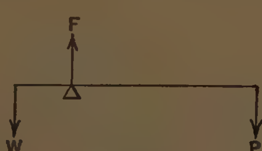


FIG. 74.

Lever of First Class.

A lever of the *second class* is shown in Figs. 75 and 76, and in this case the weight W is between the fulcrum F and the force P.

Figs. 77 and 78 show a lever of the *third class,* in which the force P is between the fulcrum F and the weight W.

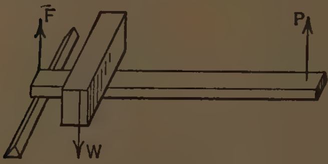


FIG. 75.

FIG. 76.

Lever of Second Class.

Principle of Levers. All problems, however complicated, dealing with levers in any form can be worked out by applying the following principle:

Weight multiplied by its distance from the fulcrum is equal to the force multiplied by its distance from the fulcrum.

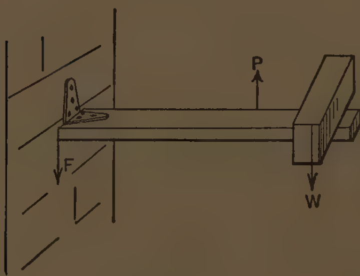


FIG. 77.

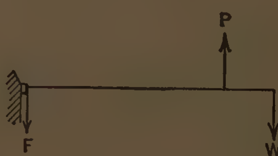


FIG. 78.

Lever of Third Class.

Problems dealing with levers are usually more easily understood when the following letters are used:

W to represent the weight,
X to represent the distance from the weight to the fulcrum,
P to represent the force,
Y to represent the distance from the force to the fulcrum.

Using these letters the principle of levers stated above is

$$W \times X = P \times Y.$$

If any three of these values are known, the fourth or "missing" value can be readily calculated by the use of one of the following equations: *

If W, P and Y are known, X may be calculated, thus,

$$X = \frac{P \times Y}{W}. \qquad (1)$$

If W, P and X are known, Y may be calculated, thus,

$$Y = \frac{W \times X}{P}. \qquad (2)$$

If W, X and Y are known, P may be calculated, thus,

$$P = \frac{W \times X}{Y}. \qquad (3)$$

If X, Y and P are known, W may be calculated, thus,

$$W = \frac{P \times Y}{X}. \qquad (4)$$

Example of Lever of First Class. If W = 100 pounds X = 2 feet and Y = 8 feet, what is the value of P?

Substituting these values in

$$P = \frac{W \times X}{Y}, \qquad P = \frac{\overset{25}{\cancel{100}} \times \cancel{2}}{\underset{4}{\cancel{8}}} = 25 \text{ pounds.}$$

This shows that for these conditions a force of 25 pounds can lift a weight of 100 pounds.

Example of Lever of Second Class. If P = 75 pounds, Y = 12 feet, and X = 2 feet, what weight W could be lifted?

* An *equation* is used to show equality; thus, $4 \times 10 = 5 \times 8$; $A = B$; $W \times X = P \times Y$ are examples of equations. These equations mean respectively that 4×10 is the same as 5×8; that A in this case has the same value as B, and that in the last equation $W \times X$ has the same value as $P \times Y$.

Substituting these values in

$$W = \frac{P \times Y}{X}, \qquad W = \frac{75 \times \overset{6}{\cancel{12}}}{\cancel{2}} = 450 \text{ pounds.}$$

A force of 75 pounds can lift a weight of 450 pounds under these conditions.

Example of Lever of Third Class. What would the distance Y be if $X = 10$ feet, $W = 20$ pounds, and $P = 50$ pounds:

Substituting these values in

$$Y = \frac{W \times X}{P}, \qquad Y = \frac{\overset{4}{\cancel{20}} \times \cancel{10}}{\underset{5}{\cancel{50}}} = 4 \text{ feet.}$$

Note the difference in efficiency between this class of lever and the preceding classes.

The lever principle is very common. When you use a pair of shears (Fig. 79), a wheelbarrow, a claw hammer in drawing nails (Fig. 80), a platform scales (Fig. 81), a screw driver you are using some form of lever. The simple crowbar is a most effective lever, and so is the handle on a hand wheel on a grindstone, or on a windlass.

Electric Power. The power developed in an electric circuit or by an electric machine is expressed in terms of watts or kilowatts. (One kilowatt = 1000 watts.) Power is determined by the following law:

Power W in watts = current C in amperes times voltage, or electromotive force, E. Or, more simply,

$$\text{Watts} = \text{Amperes} \times \text{Volts}.$$

It is customary to use the following notation in this formula:

$$W = \text{watts},$$
$$C = \text{amperes},$$
$$E = \text{volts}.$$

Fig. 79. Shears. Lever Principle.

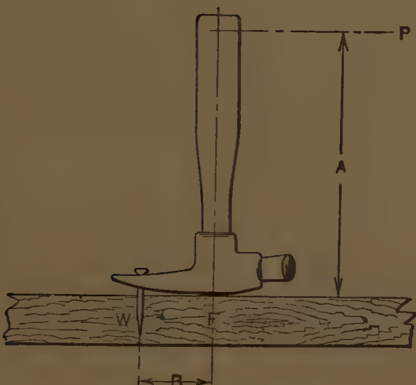


Fig. 80. Claw Hammer.

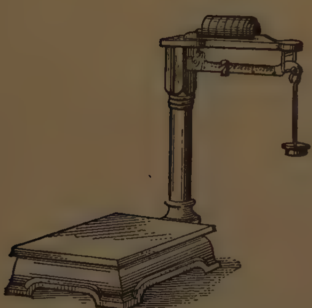


Fig. 81. Platform Scales

The formula may then be written,

$$W = CE. \tag{5}$$

From this formula you can obtain the two following:

$$C = \frac{W}{E}. \tag{6}$$

$$E = \frac{W}{C}. \tag{7}$$

Example. If a dynamo supplies 250 lamps at 120 volts, each taking .4 of an ampere, calculate the power used.

Solution. The value of C in this case is $250 \times .4$ or 100 amperes, and the value of $E = 120$ volts.

Since $$W = CE,$$

$$W = 100 \times 120 = 12{,}000 \text{ watts.}$$

Since there are 1000 watts in a kilowatt (kw.), and since it is customary to express power in kilowatts, the answer in this case is 12 kilowatts.

In the figure (Fig. 82) the ammeter A measures the current flowing through the lamp.

FIG. 82. Electrician's Ammeter.

Electrical Resistance. It is a well-known principle that the voltage E in an electric circuit is equal to the current C times the *resistance* R, measured in *ohms*. The above equation may then be written

$$\textit{Volts} = \textit{Amperes} \times \textit{Ohms}.$$

If voltage is represented by E, current by C and resistance by R, we have

$$E = CR. \tag{8}$$

From this obtain these two formulas:

$$C = \frac{E}{R}, \tag{9}$$

$$R = \frac{E}{C}. \tag{10}$$

The following illustrative problems show how the preceding formulas are applied.

Example. An electric soldering iron takes 1.3 amperes when used on a circuit of 110 volts. What is the resistance of soldering iron?

Solution. In this case, solve for R, thus,

$$R = \frac{E}{C}.$$

Therefore, $$R = \frac{110}{1.3} = 84.6 \text{ ohms.}$$

Example. What voltage is necessary to carry 1800 amperes through a resistance of .056 ohm?

Solution. Here we solve for the voltage, thus,

$$E = CR.$$

Then, $$E = 1800 \times .056 = 100.8 \text{ volts.}$$

Example. A carbon filament lamp made to burn on a 220-volt circuit has a resistance of 410 ohms. What current does it take?

Solution. Using $C = \frac{E}{R}$, we have,

$$C = \frac{220}{410} = \frac{22}{41} = .537 \text{ ampere.}$$

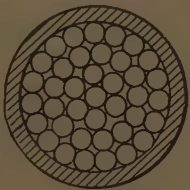


FIG. 83. Wire Conduit.

Electric Conduits. The size of the pipe or conduit required to hold a number of electric wires (Fig. 83) of any diameter may be determined by using the following approximate equation:

$$D = d \times \left\{0.94 + \frac{\sqrt{N} - 3.7}{.907}\right\}, \tag{11}$$

where D = inside diameter of pipe in inches,
d = diameter of wire in inches (including any coating, covering or insulation),
N = number of wires.

Example. How large a pipe or conduit is required to hold 25 wires which are .065 inch in diameter?

Solution. In this problem $d = .065$ inch and $N = 25$ wires. Substituting these values.

$$D = .065 \times \left\{0.94 \frac{\sqrt{25 - 3.7}}{.907}\right\},$$

$$D = .065 \times (0.94 + \sqrt{23.48}).$$

The square root of 23.48 is (4.84), so that

$$D = .065\ (.94 + 4.84 = 5.78),$$
$$D = .065 \times 5.78,$$
$$D = .3757 \text{ inch.}$$

Belting. The horse power ($h.p.$) transmitted by a belt may be determined by using the following equation:

$$h.p. = \frac{v \times e}{33{,}000},$$

where e = the effective pull on the belt in pounds,
v = the velocity of the belt in feet per minute.

Or, to express these values in other terms, we may say that

e = the force in pounds

and v = the distance through which the force acts in one minute.

The following equations to determine the values of v and e are obtained from

$$h.p. = \frac{v \times e}{33{,}000}, \tag{12}$$

$$v = \frac{33{,}000 \times h.p.}{e}, \tag{13}$$

$$e = \frac{33{,}000 \times h.p.}{v}. \tag{14}$$

Example. If a belt under an effective pull of 300 pounds is running at the rate of 1500 feet per minute, what is the horse power of the belt?

Substituting in the first of these equations,

$$h.p. = \frac{150\cancel{0} \times \cancel{300}}{\underset{11}{\cancel{33,000}}} = \frac{150}{11} = 13.6.$$

Shafting. What is the thickness of a feather key required for fastening a pulley to a shaft 2 inches in diameter?

Note: Feather keys are used so that pulleys can slide along a shaft and turn with the shaft. The key is usually fastened

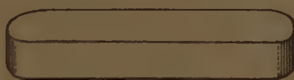


FIG. 84. Feather Key.

in the sliding pulley and is square in cross-section. Fig. 84 shows the shape of this type of key which is in most general use.

To solve the above problem we use the formula,

$$t = b = \frac{d}{4}. \tag{15}$$

t = thickness of key in inches,
b = breadth of key in inches,
d = diameter of shaft in inches.

Substituting the given values in the formula, $t = \frac{d}{4}$, we have

$$t = \tfrac{2}{4} = \tfrac{1}{2}.$$

That is, the thickness required is $\frac{1}{2}$ inch.

Cutting Speed at which any piece of work revolves in a lathe is the distance that any point on the work travels in *feet per minute.*

An approximate equation used to determine the cutting speed is

$$c = \frac{rD}{4}. \tag{16}$$

In this equation, c = cutting speed in feet per minute,
r = r.p.m. of the work,
D = diameter of work in inches.

Study this formula carefully. By means of the following equations the value of D is found when c and r are given, and likewise r is found when c and D are given; thus,

$$D = \frac{4 \times c}{r}, \tag{17}$$

$$r = \frac{4 \times c}{D}. \tag{18}$$

Note that the value of c may be obtained from the following table of approximate cutting speeds:

Soft brass,	80 feet per minute,
Gray iron castings,	40 feet per minute,
Machinery steel,	30 feet per minute,
Annealed tool steel,	20 feet per minute.

Example. What is the approximate r.p.m. for turning a chip on a tool steel reamer having a diameter of $1\frac{1}{2}$ inches?

Solution. Using formula (18),

$$r = \frac{4 \times 20}{1\frac{1}{2}} = \frac{80}{1\frac{1}{2}} = \frac{80}{\frac{3}{2}} = 80 \times \frac{2}{3} = \frac{160}{3} = 53+ \text{ r.p.m.}$$

Students should be able to solve the various quantities in a formula if it is to prove of any value. The following are important applications of commonly used formulas.

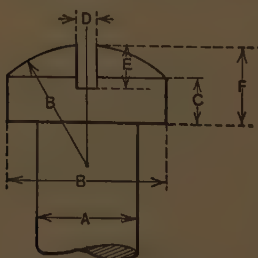


FIG. 85. Oval Head Machine Screw

Fig. 85 is an illustration of the head of a machine screw.

A = diameter of body,

$B = 1.64\,A - .009$ = diameter of head and radius for oval,

$C = .66\,A - .002$ = height of side,

$D = 1.73\,A + .015$,

$E = \frac{1}{2}\,F$ = depth of slot,

$F = .134\,B + C$ = height of head.

Suppose that A was given as .125 inch. How would the other dimensions be found? Remember that the value of A must be *substituted* in the various formulas to obtain the other dimensions?

For example,

$$B = (1.64 \times .125) - .009 = .196 \text{ inch},$$
$$C = (\ .66 \times .125) - .002 = .0805 \text{ inch},$$
$$D = (1.73 \times .125) + .015 = .231 \text{ inch},$$
$$F = 1.34\, B + C.$$

Note that .125 inch has been substituted for A in all of these formulas.

(Here the value of B (.196) and the value of C (.0805) must be substituted.) We then have

$$F = (1.34 \times .196) + .0805.$$

Therefore, $$F = .343 \text{ inch},$$

$$E = \tfrac{1}{2} F.$$

Therefore, $$E = \frac{.343}{2} = .171 \text{ inch}.$$

Summary. At this point consider what information you have obtained from this chapter. You have learned how to apply several formulas which may be classified as follows:

1. The general law of levers,

$$W \times X = P \times Y.$$

2. The formula used to determine the power developed in an electric current or by an electric machine,

$$W = C \times E.$$

3. The formula used to determine the voltage E of an electric current C flowing through a lamp,

$$E = C \times R.$$

4. The formula used to determine the diameter of the pipe required to hold a number of wires of any diameter,

$$D = d \times \left(0.94 + \frac{\sqrt{N} - 3.7}{.907}\right).$$

5. The formula used to determine cutting speed,

$$c = \frac{r \times D}{4}.$$

6. The formula used to determine the horsepower transmitted by a belt,

$$h.p. = \frac{v \times e}{33,000}.$$

Problems — Groups I and II

1. When a weight of 685 pounds is $2\frac{3}{4}$ inches from the fulcrum and can be balanced on a bar by a pull of 75 pounds, what distance is the force from the fulcrum? (See Fig. 86.)

Fig. 86. Lever Problem.

2. What force 10 feet from the fulcrum will just raise a weight of 2465 pounds 8 inches from the fulcrum with a lever of the second class?

3. How large a pipe is required to hold 32 wires .162 inch in diameter?

4. If the power of a generator is 15.6 kilowatts and the current in amperes is 48, what is the voltage?

5. A dynamo furnishes power for the following equipment at 115 volts:

20 arc lights of 10 amperes each,
500 incandescent lights of 0.55 ampere each,
12 electric heaters taking a total of 24 amperes.

Compute the load on the dynamo in kilowatts.

6. A belt under an effective pull of 420 pounds is driven by a pulley whose rim velocity is 1300 feet per minute. Compute the horse power.

7. An electric light and power company uses a belt 70 inches wide which transmits 1400 horse power at a speed of 5642 feet per minute. Compute the effective pull per inch of width. The total value of e obtained represents the total pull. Find the pull per inch of width.

8. We use ordinarily two kinds of thermometers: the Fahrenheit and Centigrade (Fig. 87). On the Fahrenheit thermometer the freezing point is 32 degrees above zero and the boiling point 212 degrees. On the Centigrade thermometer the freezing point is 0 and the boiling point 100 degrees.

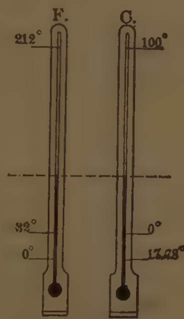


FIG. 87. Fahrenheit and Centigrade Thermometers.

The following formulas are used to change a Fahrenheit reading to a Centigrade reading and vice versa.

$$C = \tfrac{5}{9} \times (\text{Fahrenheit reading} - 32 \text{ degrees}),$$
$$F = (\tfrac{9}{5} \times \text{Centigrade reading}) + 32 \text{ degrees}.$$

If the Fahrenheit thermometer reads 57 degrees above zero, what is the corresponding reading on the Centigrade thermometer?

9. If the Centigrade thermometer reads 18 degrees above its zero what is the corresponding temperature on the Fahrenheit thermometer?

10. The efficiency of a machine is expressed in per cent. No machine has as yet been invented which is 100 per cent efficient. In the form of an equation, efficiency is stated

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}.$$

That is, the efficiency of a machine equals what we get out of it divided by what we put into it.

If a 5-horse-power electric crane lifts a weight at such a rate of speed as to be equivalent to 4.3 horse power what is its efficiency?

11. What is the efficiency of an engine if the indicated horse

power is 225 and the brake horse power 195? (The brake horse power is what the engine is actually capable of producing.)

12. Fig. 88 illustrates a wood screw.

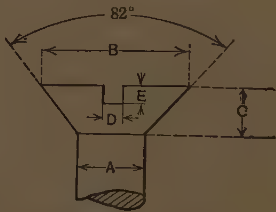


FIG. 88. Wood Screw.

$$A = \text{Diameter of the body,}$$
$$B = 2A - .008 = \text{diameter of head,}$$
$$C = \frac{A - .008}{1.739} = \text{depth of head,}$$
$$D = .173A + .015 = \text{width of slot,}$$
$$E = \tfrac{1}{3}C = \text{depth of slot.}$$

If A = .32 inch, what are the other dimensions?

13. Good proportions for a tool handle, in Fig. 89, when $H = 3\frac{3}{4}$ inches, are

$A = 1$ inch, $B = \frac{7}{16}$ inch, $C = \frac{1}{2}$ inch, $D = \frac{5}{16}$ inch, $E = 2\frac{3}{8}$ inch. $F = \frac{5}{8}$ inch, $G = \frac{3}{4}$ inch.

Make a sketch of a handle using these dimensions.

14. The average electrical resistance of the human body is approximately 10,000 ohms. If .1 ampere of sufficiently high volt-

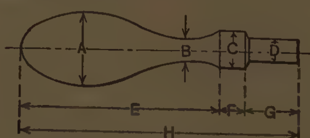


FIG. 89. Tool Handle.

age passes through the body the result is usually fatal. What is the lowest voltage at .1 ampere that would kill a person whose body had a resistance of 100,000 ohms?

15. What does it cost to run an electric car (Fig. 89*a*) a mile if a current of 60 amperes at 550 volts drives the car at an average rate of 15 miles per hour. (Electricity to cost 4 cents a kilowatt per hour.)

16. The strength of a hemp rope per square inch of area may be determined by the formula $W = S \times A$, in which W = strength

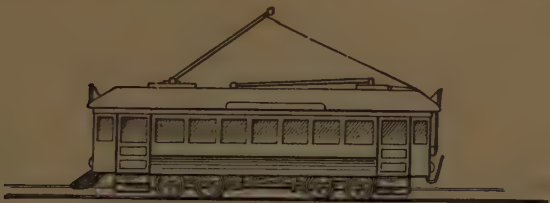


FIG. 89*a*. Electric Railway Car.

per square inch of area, S = a constant quantity (1420) and A = area of rope in square inches. This formula may be written

$W = 1420 \times .7854 \times D^2$ (D equals the diameter of the rope).

What is the strength of a hemp rope $\frac{3}{4}$ inch in diameter?

17. The formula given in problem 17 may be used to determine the diameter of a rope necessary to support a certain weight.

If $W = 1420 \times .7854 \times D^2$,

$$D^2 = \frac{W}{1420 \times .7854} \text{ or, after taking the square root,}$$

$$D = \sqrt{\frac{W}{1420 \times .7854}}.$$

What diameter of hemp rope is necessary to support a weight of 300 pounds per square inch of area?

FIG. 90. Welded Chain.

18. The strength per square inch of cross section of an electrically welded steel chain may be determined by the formula $W = 14{,}000\ D^2$, in which D equals the diameter of the bar from which the chain is made. What is the strength of a chain in which $D = \frac{3}{8}$ of an inch?

19. Fig. 91 shows an electrician's cutting pliers in about the position for cutting wire. If the pressure of the hand on *each*

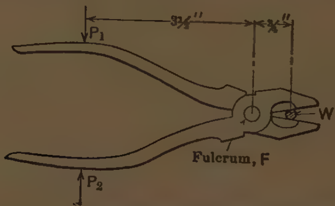


FIG. 91. Electrician's Cutting Pliers.

lever arm of the pliers (at P_1 and P_2) is five pounds, with the dimensions as shown, what is the pressure in pounds exerted on the wire at W? This is a simple lever problem.

CHAPTER IX

PRACTICAL APPLICATIONS OF GEOMETRY

You need not study all of Plane Geometry in order to find the areas of common surfaces. Plane (flat) surfaces are so common about us that we solve for areas without thinking of geometry at all. Still you should study this chapter as an introduction to the subject of *Practical Geometry.*

Surfaces are enclosed by lines. When the enclosing lines are straight, it takes at least *three* straight lines to enclose a surface. In Fig. 92, for example, no surface is en-

FIG. 92. Angle.

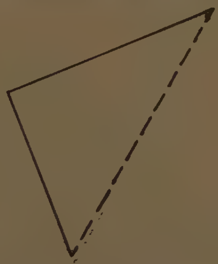


FIG. 93. Triangle.

closed and we have a single angle, but when a dotted line is added as indicated in Fig. 93 a surface is enclosed. Any surface enclosed by three straight lines is called a *triangle.*

The deck scraper shown in Fig. 94 is a good practical illustration of the surface of a triangle. By using this shape of surface it is possible to have three working edges on the same tool.

Fig. 94. Triangular Deck Scraper.

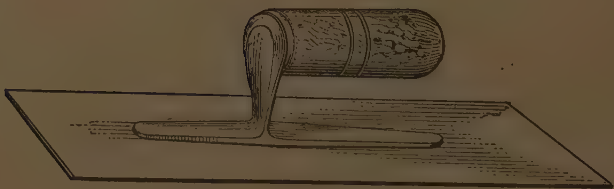


Fig. 95. Plasterer's Trowel.

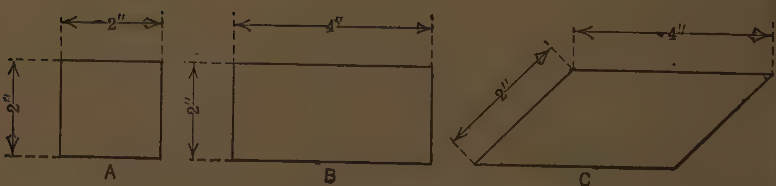


Fig. 96. Four-sided Figures.

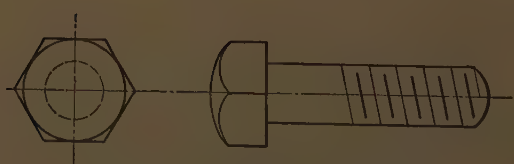


Fig. 97. Hexagonal Bolt Head.

The most common surfaces with four sides are the square and the rectangle. Objects having this shape are very numerous. The tops of many tables, the side of a room, the cover of a book, a plasterer's trowel (Fig. 95) are good illustrations of a rectangle.

A *square* is a four-sided surface that has all of its sides equal. For example in A (Fig. 96) note that all sides are 2 inches in length, while in B the opposite sides are equal and the length greater than the width. C is also a four-sided figure.

In both A and B the sides are perpendicular to each other, forming right angles; in C they are not. C is called a *parallelogram.*

There are many figures enclosed by *more than four sides* but we shall consider only the *hexagon* here because that is the only one containing more than four sides which is extensively used. A hexagon has *six* sides. Bolt heads (Fig. 97) and nuts are often made in this shape.

The figures or surfaces enclosed by straight lines, which you ordinarily use in practical work, are, therefore, the triangle, the square, the rectangle, the parallelogram and the hexagon. The only flat surface which we shall discuss here enclosed by a curve is the *circle.* A cross section in the shape of a circle like Fig. 83, page 107, is called a circular cross section. An emery wheel (Fig. 98) has a circular form; also the washers on the side of the wheel. A hexagonal nut holds on the washers. The wire gage (Fig. 99) is an illustration of the use of a modified circular form in a measuring instrument.

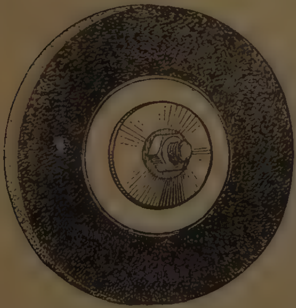


FIG. 98. Emery Wheel.

If you examine any machine composed of a considerable number of parts you would be surprised at the large number of circular shapes. Pulleys, gears, spindles, shafts, handles, etc., are usually either circular in form or circular in cross section.

FIG. 99. B. & S. Wire Gage.

In architectural work circular shapes are numerous. A good example is the porch column (Fig. 100).

FIG. 100. Porch Column.

Area of a Surface. Now that you have learned to recognize the surfaces just described, the next point to take up is the method of finding the areas of these surfaces. These will be considered in the following order: square, rectangle, parallelogram, triangle, hexagon, and circle.

Area of a Square. Suppose we wish to find the total area of the sides of the box shown in Fig. 101. Each side of this box is in the form of a square. Since the *area of a square equals the square of its side,* the area of each side of the box is 9 square inches (3 × 3). Since there are four sides the total area is 36 square inches (4 × 9). If the area of the bottom is included an additional 9 square inches must be added.

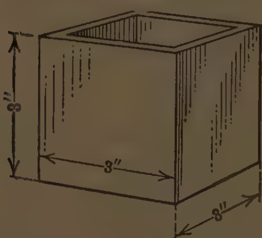


FIG. 101. Surface Measurement (of Squares).

The area of a rectangle is equal to the product of its length multiplied by its breadth. If two sides of a rectangle are 10 inches and the other sides are 5 inches, the area is 50 square

inches. Notice that when multiplied together length and breadth must both be expressed in the same denomination, as, for example, in feet or in inches. Suppose the surface area of the piece of wood illustrated in Fig. 102 is desired. All of the surfaces here are rectangles.

There are two rectangular areas at the ends of which the length is 4 inches and the width 2 inches. Each end con-

FIG. 102. Surface Measurements.

tains, therefore, 8 square inches or a total of 16 square inches for both ends. The top and bottom each contain 60 square inches (2 × 30). This makes a total of 120 square inches for the top and bottom. Each side has an area of 120 square inches (4 × 30) or a total of 240 square inches for both sides. The entire area of the surface of the piece of wood in the figure is 16 + 120 + 240 or 376 square inches.

Area of a Parallelogram. The reason that the parallelogram is mentioned here is because it is similar to the rectangle and, therefore, you may become confused in the methods for finding the areas of each. The rectangle is a parallelogram but when we think of a parallelogram it is almost always a figure shaped as illustrated in

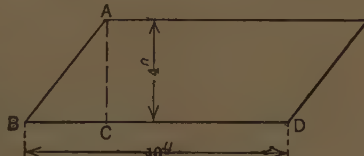


FIG. 103. Parallelogram.

Fig. 103. *The area of a parallelogram equals the product of a base by the altitude.* *AC* is the altitude and *BD* is

the base, so the area of the surface is 4×10 or 40 square inches.

The possibility of error here is that of multiplying the length of AB by the length of BD. This is incorrect because AB is not the altitude. The altitude is the *perpendicular* distance between two parallel sides.

Area of a Triangle. The next surface to consider is the triangle. The area of a triangle may be found in several ways, according to the kind of triangle that you are considering and according to the values which are given. The shaded areas in Fig. 104 represent triangular surfaces. *The*

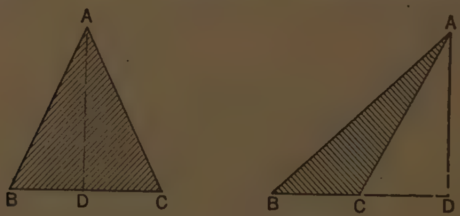


FIG. 104. Area of Triangles.

area of any one of these is equal to the product of one half the base by the altitude. The altitude of a triangle is always the perpendicular distance from a *vertex* (any of the points where the sides of a triangle intersect is a vertex) to the opposite side — called the *base*. This means that in either of the figures illustrated the area is $\frac{1}{2}(AD \times BC)$. If BC is 5 inches long and AD 6 inches long, the area is 15 square inches $\frac{1}{2}(5 \times 6) = \frac{30}{2} = 15$. Note particularly that the altitude of a triangle *may be* outside of the triangle itself as is shown in the second triangle in Fig. 104.

Area of Equilateral Triangle. There are other ways of finding the areas of triangles which you should understand. Suppose you wish to find the area of a triangular piece of glass (Fig. 105) 7 inches on the side. You

can see that the altitude is not given here but *all of the sides are of the same length.* This is called an *equilateral triangle.*

To find the area, square the side, divide the result by 4 and multiply by the square root of 3 (1.732). The area when a side equals 7 inches is

$$\begin{aligned} \text{Area} &= \tfrac{49}{4}\,(1.732).^{*} \\ &= 84.868 \text{ square inches.} \end{aligned}$$

The formula to use in a case like this is

$$\text{Area} = \tfrac{1}{4}\,(\text{side})^2\,\sqrt{3}.$$

This formula may be written

$$\text{Area} = .433\,(\text{side})^2$$

(Note that .433 is obtained by dividing 1.732 by 4.)

Area of Other Triangles. Often we know the length of the sides of a triangle but not the other dimensions. The triangle in Fig. 106 is an instance. Here the sides of the triangle are unequal but their lengths are known.

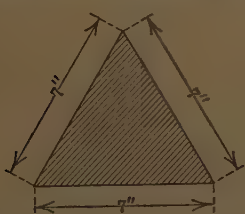


FIG. 105. Equilateral Triangle.

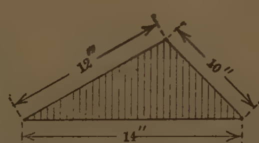


FIG. 106. Area of Triangle.

The formula to be used in finding the area of such triangles is as follows:

$$\text{Area} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}.$$

* It is easy to remember the square root of 3 as it has the same figures as the year in which Washington was born.

In Fig. 106 s = half the sum of the sides; that is, $s = \frac{1}{2}(14 + 12 + 10) = \frac{1}{2}$ of $36 = 18$.

$$a = \text{side of 14 inches.}$$
$$b = \text{side of 12 inches.}$$
$$c = \text{side of 10 inches.}$$

Then
$$s - a = 18 - 14 = 4.$$
$$s - b = 18 - 12 = 6.$$
$$s - c = 18 - 10 = 8.$$

Now, turning to the formula, we find that

$$\sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{18 \times 4 \times 6 \times 8.}$$

Then Area $= \sqrt{2456} = 49.55$ square inches.

The next surface to be considered is the regular hexagon, that is, a hexagon with all its sides equal. A regular hexagon (Fig. 107) may be divided into six equilateral triangles.

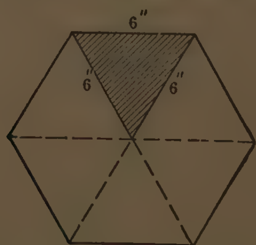


Fig. 107. Area of a Hexagon.

Since the area of an equilateral triangle equals $\frac{1}{4}(\text{side})^2 \times \sqrt{3}$ and there are *six* equilateral triangles in a hexagon, the area of the hexagon will be equal to six times the area of one of the triangles.

That is, the area of the hexagon will equal $\frac{6}{4}(\text{side})^2 \times \sqrt{3}$. This may be reduced to the more simple formula

$$\text{Area of hexagon } \frac{6}{4} \times 1.732 \times (\text{side})^2 = 2.598\,(\text{side})^2.$$

In Fig. 107 the area will equal 2.6 × 6 × 6 or 93.6 square inches.

Area of Any Polygon. A surface that is enclosed by three or more straight lines is called a polygon. We may call triangles, squares, etc., polygons. The values given in the following table may be used for finding the area of *any regular* (*all sides equal*) polygon. The area may be found by multiplying the square of one side by the constant given in the table.

Name of polygon	Number of sides	Constant for area
Triangle	3	.433
Square	4	1.000
Pentagon	5	1.720
Hexagon	6	2.598
Heptagon	7	3.364
Octagon	8	4.428
Nonagon	9	6.182
Decagon	10	7.366
Undecagon	11	9.366
Dodecagon	12	11.196

Area of a circle is found by multiplying the square of its diameter by .7854. This may be written in the form of

$$\text{Area} = .7854\,D^2,$$

where D is the diameter of the circle.

The area of a circle of which the diameter is 10 inches will equal .7854 × 100 = 78.54 square inches.

Area of Surface of a Cylinder. An interesting surface to consider because of its close relation to the circle is the so-called cylindrical surface. It is mentioned here because such a surface is really a rectangle.

Consider a hot water tank such as is used in residences. These tanks are made by first cutting out a rectangular shaped piece of metal and simply folding it around until its

edge assumes a circular shape. To find the area of a cylindrical surface, find the distance around the circle and multiply this distance by the height. If the diameter of the tank (Fig. 108) is 20 inches and the height 5 feet, the area

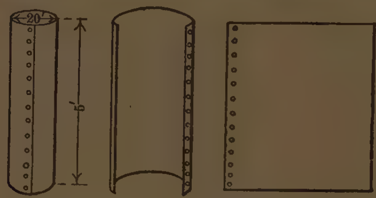


FIG. 108. Area of Surface of a Cylinder.

of the piece of metal necessary to build the tank may be found by the formula

$$\text{Area} = 3.1416 \times \text{diam.} \times \text{height}.$$

In Fig. 108, the area will equal $3.1416 \times 20 \times 60$ or 3769.92 inches.

There are two other areas which have not as yet been mentioned which you should know about, although they are not so common as those previously mentioned, especially in practical work. These areas are the trapezoid and the trapezium.

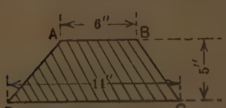


FIG. 109. Area of Trapezoid.

Area of a Trapezoid. The trapezoid is a surface (Fig. 109) bounded by four lines *two of which are not parallel.* The area of such a surface is found by multiplying the sum of the parallel sides by one half the altitude. The area of the trapezoid (Fig. 109) equals $2\frac{1}{2} \times (6 + 4)$ or 50 square inches.

Area of Irregular Surfaces. A lot of land of irregular shape is shown in Fig. 110. The area of such a surface can be found by *dividing it into triangles.* With the general formula for the area of any triangle, it is possible to find the

area of such a figure with considerable accuracy. This formula, as explained on page 124, is $\sqrt{S\ (s-a)\ (s-b)\ (s-c)}$. If a line be drawn from B to D and this line measured the lengths of the sides of the triangle will be known. Using the formula the area of both triangles may be determined. Adding these areas together we, of course, have the area of the entire figure.

The method for finding the area of an irregular surface such as Fig. 111 represents should be helpful to you. The

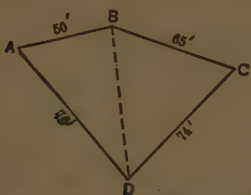


FIG. 110. Area of Surface.

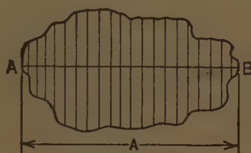


FIG. 111
Area of Irregular Surface.

surface should be drawn to scale and divided horizontally into equal sections. Measure the lengths of the vertical lines. Add these lengths and divide their sum by the number of the lines drawn. The result is the average length of the vertical lines. To find the area compute the *average* length of the vertical lines and multiply by the distance A. Do you see the relation to the method for finding the area of a rectangle? What you are really doing is changing the figure into an equivalent rectangle.

Board Measure. In measuring lumber the standard used is the *board foot.* A board foot is a piece of lumber one foot long, one foot wide and one inch thick. We think of it as being a foot square and having one inch of thickness. You will see that a board foot contains $\frac{1}{12}$ cubic foot (1 ft. $\times$ 1 ft. $\times$ $\frac{1}{12}$ ft. = $\frac{1}{12}$ cu. ft.).

To find the number of board feet in a piece of lumber, multiply the length in feet, by the width in feet, by the

thickness in *inches*. If the width should be given in inches, change it to feet by dividing by 12. For example: a plank 12 feet long, 9 inches wide and 2 inches thick contains $12 \times \frac{9}{12} \times 2$, or 18 board feet.

Lumber is usually bought and sold by the thousand feet (*M. B. F.* or simply *M.*), and comes in lengths of an even number of feet, as 10, 12, 14, 16, etc. If the thickness is less than one inch, it is counted as one inch thick in figuring the cost.

Example. What will be the cost of 19 planks 16 feet long, 14 inches wide, and 3 inches thick, at $30 per *M. B. F.*?

$$\overset{4}{\cancel{16}} \times \frac{14}{\cancel{12}} \times \cancel{3} = 56 \text{ board feet in one plank.}$$

$$\begin{array}{r} 56 \\ \underline{19} \\ 494 \\ \underline{56} \\ 1054 \end{array} \text{ board feet in the 19 planks.}$$

If 1000 board feet cost $30, 1054 board feet will cost $\frac{1054}{1000}$ or 1.054 times $30, which amounts to $31.62.

Problems — Group I

1. Angles are much used in the construction of buildings. What is the area of the section shown in Fig. 112 if the metal is $\frac{1}{2}$ inch thick?

2. A brass cover plate 8 inches in diameter has four $\frac{3}{4}$-inch holes drilled in it. Find the area of the brass in the plate after the holes are drilled. See Fig. 113.

3. How many square feet are there in a cement sidewalk 160 feet long and 4 feet 6 inches wide?

4. A house is 40 feet long, 32 feet wide and it has an average height of 19 feet. Allowing 28 square yards for gables and dormer windows, how many square yards of painted surface are there on the house?

5. If a gallon of paint covers 90 square yards in two coats, how many gallons of paint are required for the house described in Problem 4?

6. How many square feet of zinc are required to line the inside of a box (Fig. 114), the inside dimensions of which are 32 inches, 11 inches and 7 inches?

7. If a pipe has an outside diameter of $2\frac{3}{16}$ inches and an inside diameter of $1\frac{7}{8}$ inches, what is its cross-sectional area?

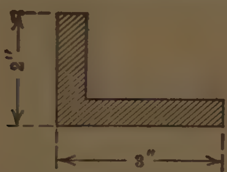


Fig. 112.
Cross-section of Steel Angle.

Fig. 113.
Brass Cover Plate.

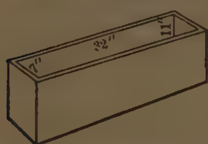


Fig. 114. Zinc-lined Box.

Fig. 115. Area of a Pond.

8. Figure 115 represents the outline of a pond as drawn on a map. Using the method described on page 127, find the approximate area

Fig. 116. Hollow Triangular Bar.

in acres. Assume that the map is *to the scale* of 1 inch equals one mile.

9. If the hollow bar shown in Fig. 116 has the dimensions indi-

cated, what is the area of the cross section if each side of the hole measures $\frac{3}{8}$ inches?

Suggestion. Draw the dotted line shown in the figure and find the altitude of the right triangle which is also the altitude of the original triangle.

10. Compute in square feet the radiating surface of a pipe radiator (Fig. 117) made of 12 pipes 2 inches in diameter, each 35 inches high.

Note. — This is equivalent to computing the outside surfaces of the pipes, which are cylindrical in shape.

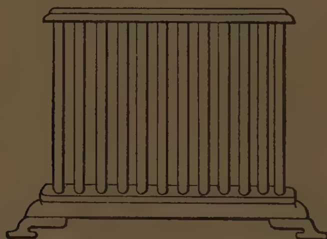


FIG. 117. Pipe Radiator.

11. A house 50 feet long and 30 feet wide is to be erected on the lot shown in Fig. 118. What per cent of the lot will the house occupy?

12. What is the area of a piece of zinc the sides of which are $8\frac{3}{8}$ inches, $6\frac{1}{8}$ inches and $10\frac{1}{8}$ inches.

13. How much will it cost to cement a floor of the dimensions shown in Fig. 119 if the cost is $2.25 per sq. yard?

14. How many linear feet of roofing will be required to cover the shed roof shown in Fig. 120 if the roofing material is 30 inches wide and 2-inch lap is allowed?

15. What is the cross-section area of the hexagonal bar shown in Fig. 121. Note that the diameter of the hole at the center is the same dimension as the side of the hexagon.

16. Since the amount of water delivered through a pipe depends upon its sectional area, how many pipes having an inside diameter

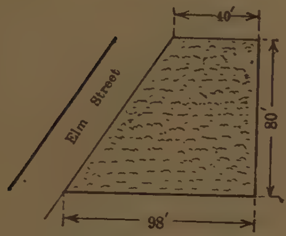


FIG. 118. Drawing of House Lot.

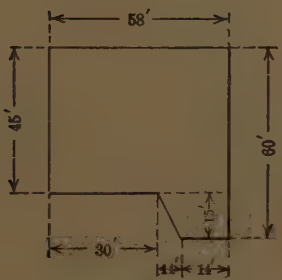


FIG. 119. Irregular Cement Floor.

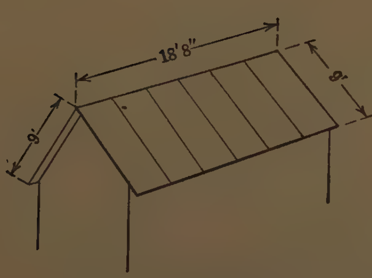


FIG. 120. Shed Roof.

of $2\frac{1}{8}$ inches will be needed to take care of what a main pipe of 8 inches inside diameter is capable of delivering?

17. Which has the greater area, a surface containing 25 square inches or a surface 25 inches square? Explain the reason for the difference and show in square inches how great it is.

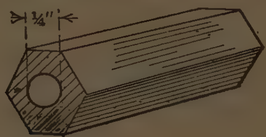


FIG. 121. Hexagonal Bar.

18. An air duct, the inside dimensions of which are 10 inches by 12 inches is to be replaced by a duct of circular cross section which must be practically the same area. What must the diameter of the circular duct be?

19. How much would it cost to paint a standpipe 60 feet high and 22 feet in diameter at .08 cent per square foot?

20. What is the sectional area in square feet of the concrete retaining wall shown in Fig. 122? Assume the width of the wall at the bottom is 24 inches.

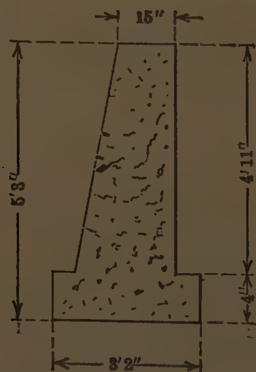


FIG. 122. Concrete Retaining Wall.

21. Find the number of board feet in a piece of timber 18 feet long, 10 inches wide and 3 inches thick.

22. (*a*) How many thousand feet of lumber are contained in a pile having 42 layers of boards 16 feet long, the width of the layers being 11 feet and the thickness of the boards 1 inch?

(*b*) What would be the cost at $23.50 per thousand board feet?

23. How many board feet in a beam 28 feet by 9 inches by 8 inches?

24. A carpenter orders the following amounts of lumber. How many *M. B. F.* are there in all?

25 pieces 12 feet by 8 inches by 1 inch.
10 pieces 20 feet by 9 inches by 8 inches.
60 pieces 8 feet by 3 inches by $\frac{1}{2}$ inch.

Problems — Group II

1. What is the area of an odd piece of sheet copper the sides of which are $3\frac{1}{6}$ feet, $27\frac{1}{3}$ inches and 23 inches?

2. Find the cross sectional area of a conduit which has an outside diameter of $5\frac{1}{2}$ inches and an inside diameter of $4\frac{5}{8}$ inches.

3. A welding machine is to be set up on a concrete foundation. The base of the machine makes a perfect rectangle of 3 feet by 5 feet. How many square inches of surface must there be on the foundation if there is to be a margin of $1\frac{1}{2}$ inches all around the base of the machine? Make a sketch of the layout of this problem before solving.

4. A circular driving shaft on a motor broke. The shaft had a diameter of $1\frac{1}{8}$ inches. When the broken ends were welded a burr having a circumference of 5.5 inches was raised about the weld. This burr had to be machined down. What was the diameter of the burr?

5. A householder desired to have the switch in his cellar placed inside an asbestos-lined box. The inside dimensions of the box were as follows: Length 12 inches, width 8 inches, depth 4 inches. How many square inches of asbestos lining were required to cover the inside of the box?

6. You are to make a circular hole, having a circumference of

12 inches, in a sheet of copper. How wide apart must be the points on the compass with which you lay out the circle?

7. The points on your compass are 5.25 inches apart. How many square inches would there be in a circular piece of sheet copper laid off with compass set as described?

CHAPTER X

PRACTICAL APPLICATIONS OF SOLID GEOMETRY

All about us there are objects illustrating the common figures of what we call solid geometry. A tool chest or an ordinary packing box is an illustration of a solid with rectangular cross section. A ball of a bearing in a bicycle wheel is a sphere. When a tinsmith makes a connection for an air duct, the article he produces probably has the shape of part of a cone. The electrician unwinding a piece of wire is handling a cylinder of very small diameter. From this you see how common geometrical solids are, and how nearly all of the objects that we use are in some way related to the rules of geometry.

Surface Area of Solids. The study of solids may well begin with those having rectangular cross sections. All the sides or faces of such a solid are rectangles. The area of its entire surface may be found by finding the areas of all the rectangles and adding them together.

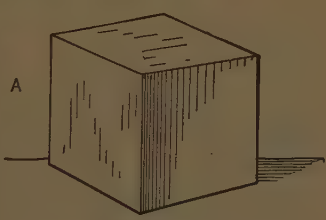


FIG. 123. Rectangular Solid.

In the solid represented in Fig. 123 there are 6 rectangular faces, each measuring 6 inches × 6 inches. The area of each face equals 6 × 6, or 36 square inches. Then the combined area of 6 faces equals

$$36 + 36 + 36 + 36 + 36 + 36 = 6 \times 36,$$

or 216 square inches.

The Volume of Solids. The volume of a solid is the quantity of space it occupies. Volume is the same as cubic contents. The measuring unit of volume is a cube whose edges are equal in length to a linear unit; for example, a cubic inch or a cubic foot. A cube is a solid whose cross sections are all squares. Its faces are, therefore, equal squares and its edges are equal in length.

Let us suppose we have a box which on the inside is 5 inches high, 3 inches wide, and 10 inches long. Now if we have a lot of 1-inch cubes we shall find that we can lay down 3 rows of 10 cubes each on the bottom of the box. This will make a layer of 30 cubes and will occupy 1 inch of the height of the box. Consequently on this we could lay 4 more similar layers, making 5 in all, just filling the box, which would then contain 5 times 30, or 150 one-inch cubes. The space occupied by a cube having its faces 1 inch square is called a cubic inch. Hence, we say this box contains 150 cubic inches.

The number of cubes in each horizontal layer is equal to the number of square inches in the base, and the number of layers is equal to the number of inches in the altitude. Therefore, the *volume of a solid with rectangular cross section may be found by multiplying the area of the base by the height.* Or we may find the volume by finding the product of the length, width and height. By this method the volume of the box is $5 \times 3 \times 10 = 150$ cubic inches.

A cube has six equal square sides. Therefore, to find the area of the outside surface of a cube, find the area of one of its square sides and multiply that area by six. Thus the area of the outside surface of the cube shown in Fig. 124 with edges measuring 7 inches is $6 \times (7 \times 7) = 294$ square inches.

The volume of a cube may be found by the same method as for any other solid with rectangular cross section. However, as the edges of a cube are all equal, the product of its

length, width and height is the same as the cube of one of its edges. Therefore we say,

$$\text{Volume of a cube} = \text{its edge cubed} = (\text{edge})^3.$$

Suppose that a box (Fig. 124) has all of its inside edges the same and that each edge measures 7 inches. The num-

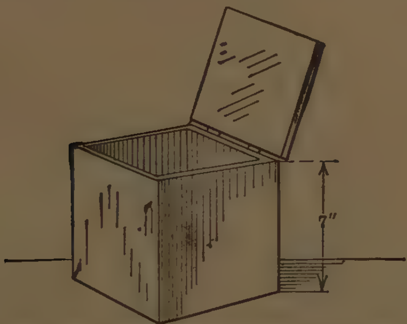


FIG. 124. Outside Dimensions of a Box.

ber of cubic inches in the box is $7^3 = 7 \times 7 \times 7 = 343$. In other words, the volume or cubic contents of the box is 343 cubic inches.

The carpenter's level (Fig. 125) is a good practical illustration of a solid with rectangular cross section. Metals

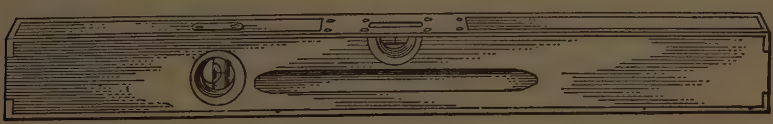


FIG. 125. Carpenter's Level.

often come in this form. The bed plates of many machines, such as lathes, frequently have rectangular cross sections.*

* Prism is the general name for solids whose ends are equal and parallel and whose sides are parallelograms. Prisms are named according to the shapes of their bases. If the base is a square the prism is called a square prism; if a hexagon, a hexagonal prism; if a triangle, a triangular prism, etc.

FIG. 126. Oilstone.

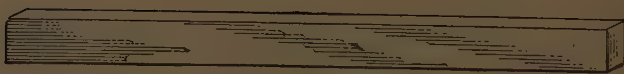


FIG. 127. Whetstone.

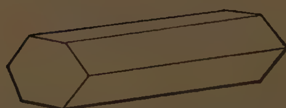


FIG. 128. Hexagonal Steel Bar.

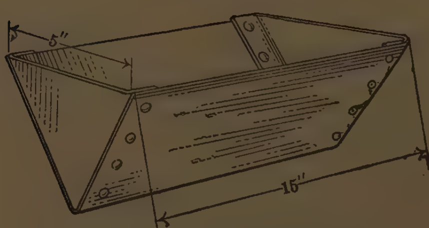


FIG. 129. Elevator Bucket.

Fig. 126 is an illustration of an oilstone in the form of a *triangular* prism.

Fig. 127 is a stone for sharpening tools which has the form of a solid with a square cross section.

Steel very often comes in the form of a solid with a *hexagonal* cross section (Fig. 128).

In this case the long faces (lateral faces) are all rectangles. To find the total areas of the surfaces of all these objects add the sum of the areas of all the lateral faces to the sum of the areas of the bases. You have learned how to find the areas of triangles and hexagons in the preceding chapter.

Finding the volume of any solid is the same as finding the cubic contents. Suppose that the elevator bucket illustrated in Fig. 129 has the cross section of an equilateral triangle, and that its length (inside) is 15 inches. The cubic contents of this bucket is determined by multiplying the area of an equilateral triangle by the length of the bucket. (The length of each side of the equilateral triangle is 5 inches and the length of the bucket is 15 inches.)

The area of an equilateral triangle is found by using the formula $\frac{1}{4}$ (side)^2 $\sqrt{3}$. As the length of a side is 5 inches, the area of the cross section equals

$$\frac{25}{4}\sqrt{3} = 6.25 \times 1.732.$$

$$6.25 \times 1.732 = 10.825 \text{ square inches.}$$

10.825 multiplied by the height of the bucket gives the volume. The volume of the bucket equals $10.625 \times 15 = 159.375$ cubic inches.

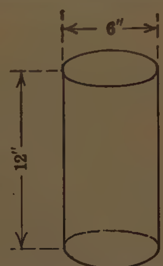


FIG. 130. Outline of a Cylinder.

Volume of Cylinders. A cylinder (Fig. 130) is similar to a solid with rectangular cross section. The surface of a cylinder, not including the base, is a rectangle. This was explained in Chapter IX, page 125. The total surface of a cylinder is

the sum of the areas of the two bases added to the area of the rectangle.

The volume of a cylinder equals the area of the base times the height. Since the base is a circle the volume may be expressed

$$\text{Volume} = .7854 \times \text{diameter}^2 \times \text{altitude}.$$

The volume of the cylinder of Fig. 130 equals $.7854 \times 6 \times 6 \times 12$, or 339.29 cubic inches.

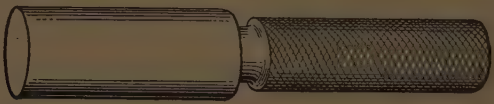


FIG. 131. Cylindrical Gauge.

Fig. 131 is a practical example of a cylinder, showing cylindrical gauge (also called plug gauge) used for measuring internal diameters.

Pyramids, Surfaces and Volume. A pyramid has only one base, and its faces (lateral, or side, faces) are triangles.

FIG. 132. Square Pyramid.

FIG. 133. Hexagonal Pyramid.

A pyramid having a square base is called a square pyramid; one with a hexagonal base is called a hexagonal pyramid, etc.

The total areas of the surfaces of pyramids include the area of the base and the sum of the areas of the triangular faces. The area of the base of a pyramid depends on the shape of that base. If the base is a triangle, the method of finding the area of a triangle must be used; if a rectangle, the method of finding the area of a rectangle, etc.

The rule generally used for determining the area of the

lateral surface, that is, the sides of the pyramid, not including the base, is as follows:

Multiply the distance around the base by one half the slant height.

The slant height is the same as the altitude of the triangles which form the faces of the pyramid AC (Fig. 134). Be sure that you understand the difference between this and the altitude of the pyramid, that is, AB. Also remember that the above rule for lateral area (that is, area of the sides) holds only when all the triangular faces have the same altitude, that is, all of the faces must have exactly the same size.

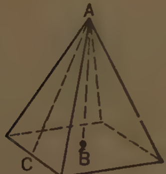


FIG. 134. Outline of a Pyramid.

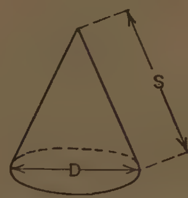


FIG. 135. Outline of a Cone.

Such a pyramid is called *regular*. If the faces are not the same the areas must be found separately and added together.

Cones. The cone is a solid whose base is circular and whose side is a curved surface which tapers to a point.

In a *regular cone* the vertex is directly above the center of the base. In this case the lateral area is equal to the distance around the base times one half the slant height, as in a pyramid. The lateral area of the cone indicated in Fig. 135 equals

$$\frac{3.1416\ DS}{2} = 1.5708\ DS,$$

that is, 3.1416 times the diameter of the base times the slant height divided by 2.

The volumes of pyramids and cones are found by multiplying the area of the base by one third of the altitude. The volume or cubic contents of the cone and pyramid

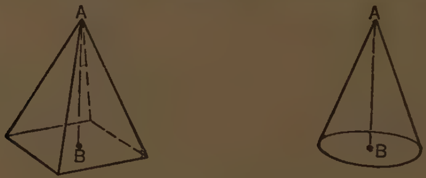


FIG. 136. Pyramid and Cone.

shown in Fig. 136 equals the area of the base times one third of the altitude (AB).

$$\text{Volume} = \text{Area of Base} \times \frac{\text{Altitude}}{3}.$$

Practical Uses of Pyramids and Cones. The plumb bob illustrated in Fig. 137 is in conical form; the end of the soldering iron (Fig. 138) is in the form of a pyramid. Church steeples are frequently built in the shape of a pyramid or cone. Emery stones, handles, files, etc., are very often molded in the form of cones and pyramids.

When a cone or a pyramid is cut by a plane parallel to the base we get what is called a frustum (Fig. 139). These shapes are extensively used in practice. The common cork stopper is a frustum.

Formula for finding the volume of a frustum of a pyramid or a cone:

$$\text{Volume} = (A + B + \sqrt{A \times B})\frac{H}{3}.$$

Here A represents the area of the upper base; B the area of the lower base; H the vertical distance between bases.

Spheres. Spheres are extensively used at the present time particularly in the form of balls for bearings. The melting ladle (Fig. 140) is an illustration of the use of half a sphere

Fig. 137. Plumb Bob.

Fig. 138. Soldering Iron.

Fig. 139. Frustums of a Cone and a Pyramid.

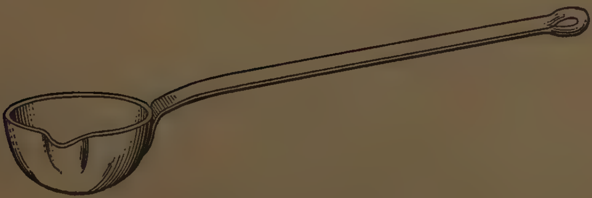


Fig. 140. Melting Ladle.

adapted to practical use. The formulas used for computing areas and volumes of spheres will be thoroughly discussed in the course in Advanced Applied Mathematics.

Illustrative Examples

The best way for you to understand the application of the above rules is to study the following illustrative problems.

Example. How large a cubical reservoir will be required to hold the water that falls on a roof, covering 500 square feet of ground, during a storm in which $\frac{5}{8}$ of an inch falls?

Solution. Here the amount of rainfall is equivalent to a solid with rectangular cross section having a base containing 500 square feet and an altitude of $\frac{5}{8}$ inch = .0521 feet.

Volume of water = 500 × .0521 = 26.05 cubic feet.

Since the volume of a cube equals the cube of its edge, the side of the reservoir necessary to hold the water will equal the cube root of 26.05. This is about 2.96. Therefore, each side of the reservoir will be 2.96 feet.

Example. If a room has the dimensions 10 feet × 12 feet × 15 feet, and 30 cubic feet of air per minute is required per person in the room, how often should the air be changed if the room accommodates 15 persons?

Solution. The volume of the room equals the product of 10, 12 and 15 or 1800 cubic feet. Fifteen persons will require 450 cubic feet of air per minute.

$$\frac{1800}{450} = 4.$$

The air should be changed at intervals of 4 minutes.

Example. A pile of coal is in the form of a cone. The diameter of the base is 35 feet; its height is 27 feet. How many pounds are there in the pile if the coal weighs 62 lbs. per cubic foot?

Solution. The volume of a cone equals the area of the base times one-third the altitude. The area of the circular base in this case is $.7854 \times 35^2 = 962.11$ square feet. The volume will be equal to 9 times 962.11 or 8658.99 cubic feet. The weight of the pile will equal 8658.99 times 62 or 536857.38 pounds.

Example. A pyramidal steel cap on the top of a water tower is 22 feet square. It has a slant height of 14 feet. What will it cost to paint it at 9 cents per square foot?

Solution. The lateral area (area of the sides) is to be found here. This equals the perimeter (distance around) of the base multiplied by one half the slant height. The lateral area will therefore be

$$\tfrac{14}{2}(22 + 22 + 22 + 22) = 7 \times 88 = 616 \text{ square feet.}$$

At 9 cents per square foot the cost of painting will be $.09 \times 616 = \$55.44$.

Example. How many cubic inches does the hexagonal bar with a circular hole, as illustrated, in Fig. 141 contain?

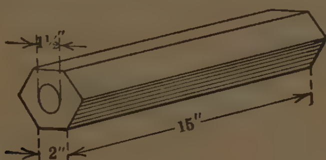


FIG. 141. Hexagonal Steel Bar

Solution. The bar is in the form of a hexagonal prism. Its volume must be found and the volume of a cylinder of the same length subtracted from it.

The volume of the prism equals the area of the hexagon (2 inches on a side) times the altitude (15 inches).

The volume of the cylindrical hole equals the area of $1\frac{1}{2}$ inches wide times the altitude (15 inches). The area of the hexagon equals $2.598\, S^2$, in which S equals a side (in this case 2 inches).

The volume of the hexagonal prism will equal

$$2.598 \times 2 \times 2 \times 15 \text{ or } 155.88 \text{ cubic inches.}$$

The volume of the cylinder equals the area of a $1\frac{1}{2}$-inch circle (1.767 square inches) times 15 or 26.505 cubic inches.

The volume of the metal remaining will equal the volume of the cylinder subtracted from the volume of the hexagon.

$$155.88 - 26.505 = 129.375 \text{ cubic inches.}$$

The following tables are given not only for use in this assignment but for any future use that you may require of them.

TABLE A

Metal or composition	Weight per cubic inch, pounds	Weight per cubic foot, pounds
Aluminum	0.0924	159.7
Antimony	0.2422	418.7
Barium	0.1354	234.0
Bismuth	0.3538	611.5
Boron	0.0939	162.2
Brass: 80 C., 2 oz.	0.3105	536.6
70 C., 3 oz.	0.3032	524.1
60 C., 4 oz.	0.3018	521.7
50 C., 5 oz.	0.2960	511.6
Bronze	0.3195	552.2
Cadmium	0.3105	536.6
Calcium	0.0567	98.0
Chromium	0.2347	405.6
Cobalt	0.3123	539.8
Copper	0.3184	550.4
Gold	0.6975	1205.6
Iridium	0.8094	1399.0
Iron, cast	0.2600	449.2
Iron, wrought	0.2834	489.8
Lead	0.4105	709.5
Magnesium	0.0628	108.6
Manganese	0.2679	463.0
Mercury (60° F.)	0.4902	847.4
Molybdenum	0.3090	534.2
Nickel	0.3177	549.1
Platinum, rolled	0.8184	1414.6
Platinum, wire	0.7595	1312.9
Potassium	0.0314	54.3
Silver	0.3802	657.1
Sodium	0.0354	61.1
Steel	0.2816	486.7
Tellurium	0.2256	390.0
Tin	0.2632	454.8
Titanium	0.1278	220.9
Tungsten	0.6776	1171.2
Vanadium	0.1986	343.2
Zinc, cast	0.2476	428.1
Zinc, rolled	0.2581	446.1

TABLE B

Substance	Weight per cubic foot, pounds	Substance	Weight per cubic foot, pounds
Asbestos	175	Gypsum	137
Asphaltum	87	Ice	56
Borax	109	Ivory	115
Brick, common	112	Limestone	163
Brick, fire	144	Marble	169
Brick, hard	125	Masonry	150
Brick, pressed	134	Mica	175
Brickwork, in mortar	100	Mortar	94
Brickwork, in cement	112	Phosphorus	112
Cement, Portland	194	Plaster of Paris	112
Chalk	163	Quartz	163
Charcoal	25	Salt, common	131
Coal, anthracite	94	Sand, dry	100
Coal, bituminous	79	Sand, wet	125
Concrete	137	Sandstone	144
Earth, loose	75	Slate	175
Earth, rammed	100	Soapstone	169
Emery	250	Soil, common black	125
Glass	163	Sulphur	125
Granite	166	Trap	187
Gravel	109	Tile	112

TABLE C

METRIC AND ENGLISH CONVERSION TABLE

Linear Measure

1 kilometer = 0.6214 mile.

1 meter = { 39.37 inches. 3.2080 feet. 1.0936 yard.

1 centimeter = 0.3937 inch.

1 millimeter = 0.03937 inch.

1 mile = 1.609 kilometer

1 yard = 0.9144 meter

1 foot = 0.3048 meter

1 foot = 304.8 millimeters

1 inch = 2.54 centimeters

1 inch = 25.4 millimeters

Square Measure

1 square kilometer = 0.3861 square mile = 247.1 acres.

1 hectare = 2.471 acres = 107,640 square feet.

1 acre = 0.0247 acres = 1076.4 square feet.

1 square meter = 10.764 square feet = 1.196 square yard.

1 square centimeter = 0.155 square inch.
1 square millimeter = 0.00155 square inch.

---

1 square mile = 2.5899 square kilometers.
1 acre = 0.4047 hectare = 40.47 acres.
1 square yard = 0.836 square meter.
1 square foot = 0.0929 square meter = 9290 square centimeters.
1 square inch = 6.452 square centimeters = 645.2 square millimeters.

Cubic Measure

1 cubic meter = 35.314 cubic feet = 1.308 cubic yard.
1 cubic meter = 264.2 U. S. gallons.
1 cubic centimeter = 0.061 cubic inch.
1 liter (cubic decimeter) = 0.0353 cubic foot = 61.023 cubic inches.
1 liter = 0.2642 U. S. gallon = 1.0567 U. S. quart.

---

1 cubic yard = 0.7645 cubic meter.
1 cubic foot = 0.02832 cubic meter = 28.317 liters.
1 cubic inch = 16.383 cubic centimeters.
1 U. S. gallon = 3.785 liters.
1 U. S. quart = 0.946 liter.

Weight

1 metric ton = 0.9842 ton (of 2240 pounds) = 2204.6 pounds.
1 kilogram = 2.2046 pounds = 35.274 ounces avoirdupois.
1 gram = 0.03216 ounce troy = 0.03527 ounce avoirdupois.
1 gram = 15.432 grains.

---

1 ton (of 2240 pounds) = 1.016 metric ton = 1016 kilograms.
1 pound = 0.4536 kilogram = 453.6 grams.
1 ounce avoirdupois = 28.35 grams.
1 ounce troy = 31.103 grams.
1 grain = 0.0648 gram.

---

1 kilogram per square millimeter = 1422.32 pounds per square inch.
1 kilogram per square centimeter = 14.223 pounds per square inch.
1 kilogram meter = 7.233 foot pounds.
1 pound per square inch = 0.0703 kilogram per square centimeter.
1 calorie (metric thermal unit) = 3.968 B.T.U. (British thermal unit).

Problems — Group I

1. A flat car with sides on is 40 feet 4 inches long, 8 feet 6 inches wide and 3 feet 4 inches deep. How many cubic yards of crushed stone does it hold when level filled?

2. There are 500 people in an auditorium, 50 feet wide, 64 feet long and 40 feet high. How many cubic feet of air are there available for each person?

3. If a brick is 8 inches × 4 inches × 2 inches how many bricks are there in a wall 120 feet long, $1\frac{1}{2}$ feet thick and 12 feet high, making no allowance for mortar?

4. To find the number of board feet in a piece of lumber or a board, multiply its length in feet by its width in feet by its thickness in inches.

How many board feet are there in a piece of redwood lumber 22 feet by 6 feet by 8 feet?

5. How many board feet are there in a wagon load of lumber 12 feet long, 3 feet 3 inches wide, and 2 feet 8 inches high?

6. The Doyle rule for finding the number of board feet in a log is extensively used in this country. It is $N = \left(\frac{d-4}{4}\right)^2 L$.

In this formula N = number of board feet,
d = diam. of small end in inches,
L = length of log in feet.

How many board feet will there be in a log 30 feet long and with a diameter of 14 inches at the small end?

7. The inside dimensions of a concrete watering trough are 8 feet by 2 feet 6 inches by 17 inches. How many gallons does it hold, allowing $7\frac{1}{2}$ gallons to a cubic foot?

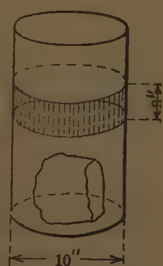


Fig. 142. Illustrating Problem 8.

8. A cylindrical vessel 10 inches in diameter is partially filled with water. If a piece of iron is dropped into the vessel, raising the water 3 inches, what is the volume of the iron?

9. How many cubic feet are there in a mile of wire $\frac{1}{16}$ inch in diameter?

10. A piece of sheet iron has the dimensions indicated in Fig. 143. If it should be folded from left to right to form the surface of a cylinder, what would the diameter of the cylinder be?

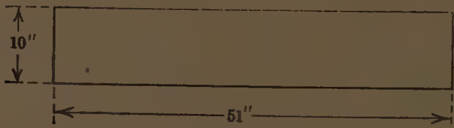


FIG. 143. Piece of Sheet Iron.

11. How many square inches are there in the grinding surface of a grindstone which is $2\frac{1}{2}$ feet in diameter and $3\frac{1}{2}$ inches thick?

12. A brass paper weight is in the form of a square pyramid. Each side of the base is 2 inches; the height is $2\frac{1}{4}$ inches. How

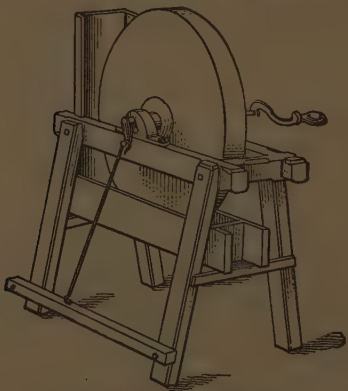


FIG. 144. Grindstone.

many pounds of brass are required for 50 such weights, assuming that brass weighs 520 pounds per cubic foot.

13. How many square yards of canvas are required for a conical tent of the dimensions shown in Fig. 145?

14. How much will it cost to paint a steeple in the form of a hexagonal pyramid if the slant height of the steeple is 45 feet and each side of the base is 5 feet 6 inches? The cost of painting is 54 cents per square yard.

15. What will it cost to construct 100 feet of a concrete wall of the cross section shown in Fig. 146 at the cost of $8.00 per cubic yard?

Note. — Remember that this is really a "solid" two feet long with a base of the section shown.

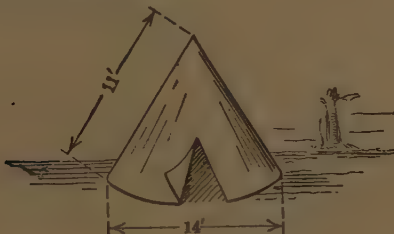


Fig. 145. Conical Tent.

16. If a cubic foot of wrought iron weighs 485 lbs., what is the weight of 500 linear feet of conduit pipe of the section shown in Fig. 147?

17. How many cubic yards are in a pile of sand 10 feet in diameter having a slant height of 8 feet? (See Fig. 148.)

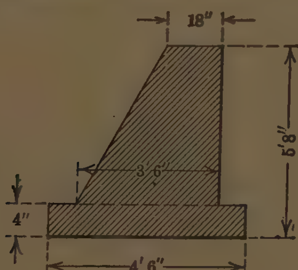


Fig. 146. Concrete Wall.

Note. — The altitude H must be found. The method of finding the side of a right triangle must be used: that is, $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$. (See page 88.)

18. What relation does the diameter of a ball or sphere have to the diameter of the largest circle that can be drawn on its surface?

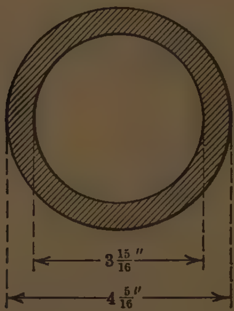


Fig. 147. Cross Section of Conduit Pipe.

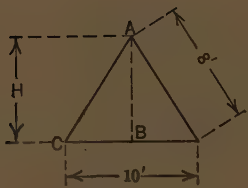


Fig. 148.
Dimensions of Sand Pile.

Fig. 149.
Illustrating Problem 18.

Fig. 150. Illustrating Problem 20.

19. How many feet of copper wire $\frac{3}{32}$ inch in diameter can be drawn from a cube of the metal measuring 6 inches on edge?

20. Using any convenient radius, draw a figure as indicated in Fig. 150. Cut it out and fold. A lap may be allowed for pasting together. What kind of a surface is obtained?

PROBLEMS — GROUP II

1. How many cubic inches of metal are there in a copper bus bar $1\frac{1}{2}$ inches wide, $\frac{5}{16}$ inch thick and 27 feet $8\frac{3}{4}$ inches long?

2. An alternating current generator (usually called an "alternator") requires a foundation 8 feet long, 7 feet 6 inches wide and 4 feet deep. How many cubic feet of concrete are there in this foundation?

3. The coal bin of a power station is 39.49 feet long, 7 feet wide and 6 feet high. How many cubic feet does it contain?

4. The air reservoir of an electric car has a diameter of 11 inches and a length of 47.53 inches. How many cubic inches does it contain?

5. An electric oven was made in the shape of a cylinder $24\frac{1}{2}$ inches in diameter and 26 inches high. What is its volume in cubic inches?

6. What is the weight of a copper bus bar 6 feet $10\frac{1}{16}$ inches long, 1 inch wide and $\frac{1}{4}$ inch thick?

7. If transformer oil weighs 5.35 pounds per gallon, what is the weight of oil contained in a cylindrical tank 1 foot in diameter and 8.22 feet high? There are 231 cubic inches in a gallon.

8. A concrete foundation for a $\frac{1}{2}$-horse-power motor is 2 feet 6.3 inches long, 1 foot 3 inches wide and 1 foot 6 inches high. How much does it weigh? (See page 147.)

9. Sometimes bus bars are made of hollow copper tubing. What would be the weight of such a bus bar 9 feet long, having an outside diameter of $\frac{3}{4}$ inch and an inside diameter of $\frac{7}{16}$ inch?

10. A motor shaft made of rolled steel is 5 feet long and weighs 194.7 pounds. What is its diameter?

11. Calculate the number of board feet in a timber 12 feet long, 15 inches wide and 3 inches thick.

12. A timber for an outdoor transformer is 16 feet long and 4 inches thick, is $7\frac{1}{2}$ inches wide at one end and $4\frac{1}{2}$ inches wide at the other end. How many board feet does this timber contain? How much will four such timbers cost at \$30 per thousand board feet?

CHAPTER XI

THE USE OF CURVES; CURVE-SHEETS AND TABLES

Tables and curves are used extensively in a great many lines of work in order to show how quantities may be compared with one another, as, for example, the weight of steel in pounds and the *corresponding* volume of steel in cubic inches. Besides being time-savers curves present data and statistics at a glance, and are therefore much easier to read than tabulated numbers. In the bulletins issued by manufacturing companies, for example, the results of tests on engines, electric generators, motors, heaters, lathes, etc., are given in the form of curves or curve-sheets, as the charts showing plotted data are called. For this reason the mechanic or engineer who wishes to be successful should be able to read and understand such curves.

The paper on which such curves are made is called "squared" paper, cross-section paper, or "plotting"

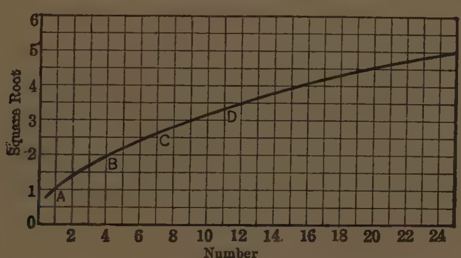


FIG. 151. Illustrations of a Plotted Curve.

paper. This paper may be procured at stationery stores, or, for rough work, may be drawn freehand. Fig. 151 represents all the essentials of a completed curve.

In this figure the point marked O is called the "origin," the word being used in the sense of starting place. The two rows of numbers or quantities, extending upward from and horizontally to the right of the origin O, give the means of locating points in the "squared" area between them. The heavy line $ABCD$ whether actually curved or straight is called the *curve*, even when under certain conditions it is a broken or zigzag line. In order that the method of plotting may be clear, let us work out the following exercise in which the square roots of certain numbers are to be determined.

Exercise. Plot a curve which will show merely by looking at the figure the square root of any number within the limits of the numbers given in the horizontal row (between 0 and 24).

Now turn to Fig. 151. We find that the square root of any number in the horizontal row is read on the vertical row opposite the point where a vertical line crosses the heavy curve at the number selected. Thus, a vertical line through 4 in the horizontal row crosses the heavy line at B and the number opposite in the vertical row is 2. Then the square root of 4 as read from the *curve* is 2. The square roots of other numbers are similarly obtained.

A curve like this is made by locating corresponding values in the horizontal and the vertical rows. Such corresponding values are given in the following table:

Number	Square root	Letter on curve
1	1.0	A
4	2.0	B
7	2.65	C
11	3.31	D

To locate the first point go from the "origin" 1 space to the right and then up 1 space. Then A will be located

at the point where the lines drawn (vertically and horizontally) from the two 1's meet.

In similar manner locate *B* by going from the origin 4 units to the right and up 2 units.

In locating point *C*, divide the space between 2.5 and 3.0 into five equal parts and estimate the position of 2.65.

After locating as many points as needed in this way, draw a smooth curve through the points.

Now, having drawn the curve, we can use it to read the squares or square roots of any values on the sheet. For example, the square root of 5 is 2.24, of 12 is 3.46, and so on. Reading the other way we find the *square* of 2 is 4, of 3.9 is 15.2, and so on.

Until thoroughly practiced you will find it helpful to use a ruler, or other straight-edge, in locating the roots or squares of numbers lying between those given along the vertical and horizontal margins.

At this point it is essential that you remember the following: *If four spaces along the horizontal row represent one unit, eight spaces represent two units, twelve spaces represent three units, etc.* The same is true in regard to the vertical row of figures.

Another problem is worked out below, showing a curve for an entirely different purpose.

An insurance company gives insurance of $100 at death for the following yearly premiums:

Age of insured	Premium
20	$2.20
25	2.40
30	2.65
35	2.95
40	3.35
45	3.90
50	4.50
55	5.85

By plotting the premiums in dollars per year as the horizontal row of figures and the ages of the insured in years as the vertical row we get a curve as shown in Fig. 152.

From this curve we can estimate very closely the premiums which should be charged for any intermediate age. The curve shows that a person 54 years old pays a premium

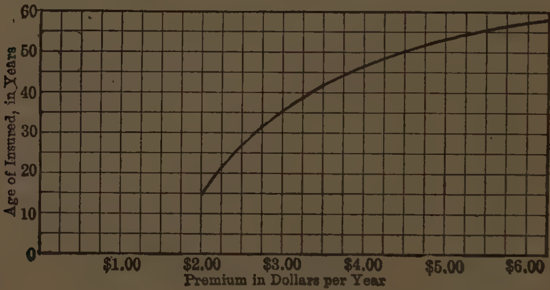


FIG. 152. Premium in Dollars per Year.

of about $5.00 per year. Notice that since we let 4 spaces equal $1.00, the $2.00 line must be 8 spaces to the right of the "origin" or zero point. The $3.00 line is 12 spaces and the $4.00 line is 16 spaces to the right of the origin.

The same is true of the vertical row of figures. If one space equals five years, all other spaces of the same size must equal five years, two spaces equal ten years, three spaces equal fifteen years, etc.

In Fig. 153 the relation between the weight of water in pounds and cubic feet of water is given. By means of this chart we can find the weight of volumes of water up to ten cubic feet. *Vice versa*, we can determine how many cubic feet of water are required to weigh from 0 to 600 pounds. The following rule is used in plotting the diagram: As 1 cubic foot of water weighs 62.5 pounds, the weight of water in pounds is equal to 62.5 times the cubic feet of water.

When locating points which appear to lie quite generally along a smooth curve, you should investigate any particular point or points which are considerably removed from this general curve. There may be an error in the original values or in marking the points. This is one way of checking the work.

Another good check is to verify the location of points by reading the horizontal values and the corresponding vertical values from the curve and checking these values with the table which was used in locating the points on the curve.

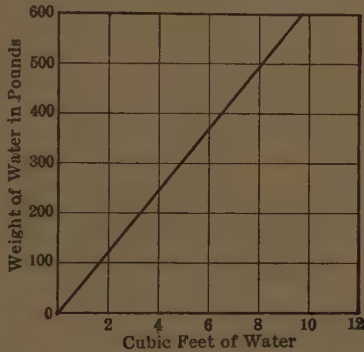


FIG. 153. Weights of Volumes of Water.

In making curves it is very necessary to label the horizontal and vertical values plainly, especially when more than one set of values are indicated on a given axis, as is often the case when more than one curve appears on the same sheet.

By looking through technical and trade books and magazines, you will see how much curves are used and how essential it is that you be able to understand them.

PROBLEMS — GROUPS I AND II

1. The voltage necessary to cause an electric spark to jump between two points or surfaces varies with the distance and depends somewhat on the shape of the surfaces. Below is a set of spark gap voltages for various distances between needle points. The distances are given in millimeters. (25.4 millimeters = 1 inch.)

Volts	Millimeters
10,000	11.9
15,000	18.4
20,000	25.4
25,000	33.0
30,000	41.0
35,000	51.0
40,000	62.0
45,000	75.0
50,000	90.0

Plot a curve using *volts* as ordinates and *millimeters* as abscissas.

2. A curve plotted from the following values will show how the voltage of a storage cell decreases with use. The storage battery used on an automobile consists of five of these cells.

Hours discharge at normal rate	Volts per cell
0	1.43
.25	1.34
.5	1.31
1.0	1.28
1.5	1.26
2.0	1.23
2.5	1.21
3.0	1.19
3.5	1.18
4.0	1.17
4.5	1.15
5.0	1.11
5.5	.92

Plot the curve using *Volts per cell* in the vertical row and *Hours discharge* in the horizontal. The curve will show very clearly how rapidly the voltage drops off after five hours of discharge.

3. The formula used to find the area of a circle when the diameter is given is as follows: The area A equals 3.1416 times the square of the radius R, or πR^2. Let the diameters represent the horizontal numbers and the corresponding areas the vertical. Plot the curve, given the following diameters: 1, 2, 4, 5 and 7 inches. From your plot estimate area of circles with diameters of 3 and 6 inches.

Horizontal	Vertical
1	.7854
2	
4	
5	
7	

etc. (complete the table)

CHAPTER XII

GENERAL REVIEW

This chapter consists of a series of problems based on the principles which you have studied during the course. You should look upon this exercise as a review. It will be to your advantage to solve the problems if possible without reference to previous assignments, although it is permissible for you to refer to them if absolutely necessary.

PROBLEMS — GROUP I

1. A screw machine makes 90 small screws per minute. If one man can look after 11 such machines how many gross of screws make up a day's work?

2. If $8\frac{2}{3}$ yards of No. 3 wire weigh one pound, compute the weight of a mile of the wire.

3. Decide which of the following boards could be cut into lengths 2 feet 8 inches long with the smallest amount of waste. The lengths of the boards are 12 feet, 13 feet and 14 feet.

4. In order to secure a speed of 8500 feet per minute does a small saw make more or fewer revolutions than a larger one? Explain fully the reasons for your answer.

5. Remembering that Watts = Amperes $\times$ Volts, what power is consumed by an arc lamp taking 5 amperes at 110 volts? (The watt is the measure of power.)

6. Using the same formula compute the voltage for which a motor is built if it is rated to take 5 amperes and to consume 1200 watts.

7. How many watts are consumed by an electric iron which uses 2.4 amperes in a 110-volt circuit?

8. A 16-inch emery wheel has a cutting speed of 4800 feet per minute. The wheel is driven by a 3-inch pulley on the same

spindle which takes a belt from a 10-inch pulley. Find the revolutions per minute of the 10-inch pulley.

9. On a simple geared lathe the threads cut per inch

$$= \frac{\text{Threads on lead screw} \times \text{teeth on lead screw gear}}{\text{Teeth on stud gear}}.$$

Using an 80-tooth gear on the lead screw and a 16-tooth gear on the stud, how many threads may be cut per inch when the lead screw has 8 threads per inch?

10. A cellar was excavated for a house 32 feet long and 28 feet wide. It had to be dug 4 feet longer and 4 feet wider to allow for building the foundation. It was dug down to an average depth of 4 feet, how many cubic feet of earth were removed?

11. Shingles are sold by the thousand. There are four bundles to the thousand. If laid 4 inches to the weather, 4 bundles will cover 100 square feet or 1 square. How much will it cost to shingle a gable roof 32 feet by 45 feet with shingles worth $8 per M. (thousand)?

12. Compute the total area of a hip roof which has a ridge 18 feet long, the side and longest rafters 15 feet long, the side eaves 38 feet long and the end eaves 22 feet long.

13. The resistance of wire to an electric current varies directly as its length. For example, the resistance of a wire 100 feet long is 100 times as great as for a wire of the same cross section, 1 foot in length. What is the resistance (in ohms) of 1000 feet of copper wire .01 inch in diameter if the resistance for 50 feet is 5.15 ohms?

14. If the resistance of one foot of wire of a certain size is .082 ohm, how many feet of wire will it take to make a resistance of 50 ohms?

15. How many square feet of radiating surface has a pipe whose outside diameter is 3 inches and length is 10 feet 6 inches?

16. What is the net cost of 15 dry batteries at $.90 less 50 per cent and 10 per cent?

17. Two pulleys in a machine shop are connected by a belt. One has a radius of 8 inches and one a radius of 2 inches. How much faster is one running than the other?

18. A concrete conduit has a cross-sectional area of 64.6 square inches. If the inside diameter is 10 inches what is the outside diameter?

19. A contractor owns a concrete mixer which cost him \$1600. It costs him \$18 a day to run it. Figuring money worth 6 per cent per year and depreciation on the machine at 20 per cent per year what is the total cost of running the machine for a year containing 200 working days?

20. An earth embankment rises 18 inches for every foot of distance on level ground. How much does the embankment rise for 18 feet of level ground?

21. A man pushes down on the end of a crowbar with a force of 110 pounds. The distance from his hand to the fulcrum is $4\frac{1}{2}$ feet and the distance from the fulcrum to the bearing point on the crowbar is 5 inches. How many pounds can the man lift?

22. Concrete for a certain job is to be mixed, 1 part cement, 3 parts sand and 5 parts gravel. If 300 bags of cement are required, how many bags of gravel and sand are needed?

23. How much does an ordinary concrete wall weigh which is 9 inches thick, 14 feet high and 32 feet long, considering that concrete weighs 137 pounds per cubic foot?

24. A piece of sheet iron contains 8281 square inches and is in the form of a square. What is the length of each side in feet and inches?

25. How many barrels of water does a circular cistern contain which is 10 feet high and 6 feet in diameter? (A square foot of water contains $7\frac{1}{2}$ gallons and a barrel contains $31\frac{1}{2}$ gallons.)

26. How many feet of wire are required for four guy wires to support the pole illustrated in Fig. 71, (see page 93) allowing 8 feet on each wire for fastening?

27. How many board feet are required for a 28-foot by 62-foot floor laid with 1-inch by 4-inch flooring, allowing 20 per cent for matching and waste?

28. How much will it cost to make the cone-shaped top of a ventilator of galvanized iron at 42 cents per square foot if the diameter is 32 inches and the height 20 inches?

29. A granite tower is built in the form of a hexagonal pyramid. Each side of the base measures 6 feet. The height is 160 feet. How many tons does the tower weigh, taking the weight of granite as 166 lbs. per cubic foot?

30. How many bricks laid on edge are required to lay a walk 20 feet long and 4 feet wide? Allow 5 per cent for waste. The dimensions of the brick are $8\frac{1}{2}$ inches $\times$ $2\frac{1}{2}$ inches $\times$ $4\frac{1}{2}$ inches.

31. A tank 3 feet square is filled to a depth of 3 feet with water. What is the pressure on the bottom of the tank?

(One gallon of water weighs $8\frac{1}{3}$ pounds. See also page 148.)

32. How large a square bar of steel can be machined from a round bar 2 inches in diameter?

33. Write out as many of the following formulas as possible without reference to any text.

Area of square =
Area of rectangle =
Area of parallelogram =
Area of triangle in terms of bases and altitude =
Area of triangle when three sides are given =
Area of equilateral triangle =
Area of regular hexagon =
Area of trapezoid =
Area of circle =
Circumference of circle =
Volume of cube =
Volume of rectangular prism =
Volume of cylinder =
Volume of pyramid =
Volume of cone =
Lateral area of pyramid =
Lateral area of cone =

You should test yourself frequently on these rules and formulas. Every practical man should have them at his "tongue's end" because of the frequency with which they

are used. If you are a poor memorizer, you should at least know where you can find them when needed.

PROBLEMS — GROUP II

1. An electric oven takes 64 amperes when connected to a 220-volt circuit. What is its resistance?

2. The resistance of a wire is proportional to its length. A wire 14 inches long has a resistance of 2.8 ohms. How long is a piece of wire which has a resistance of 9.6 ohms?

3. The voltage of a generator is proportional to its speed within certain limits. When running at 190 revolutions per minute, a generator supplies power at 95 volts. At what speed must it run to give 105 volts?

4. A motor connected to a 220-volt circuit takes 60 amperes. How many horse power does it require?

5. Electric railway motors operate on 550 volts. If the air compressor motor requires 16.8 amperes, how many watts does it use?

Electric power is measured in watts. Watt = volt × ampere. Refer to page 104 for formula. One horse power is equal to 746 watts. This gives another equation:

$$\text{Horse power} = \frac{\text{Volt} \times \text{Ampere}}{746}.$$

6. The power consumed by tungsten lamps is usually stated on a label found on the lamp. The total power required for a number of lamps is obtained by adding the amounts stated on the labels.

To a 110-volt circuit are connected tungsten lamps as follows: Two 15-watt, twelve 25-watt, sixty 40-watt and four 250-watt. How many amperes are required for this circuit?

7. A motor makes 1800 revolutions per minute when the scales indicate $16\frac{2}{3}$ pounds. The brake arm is 6 feet long. How many horse power does the motor deliver?

8. In order to estimate the cost of electroplating, the surface is calculated. How many square inches area are there on the top of a flat-headed hexagonal bolt which is 0.5 inch on a side?

9. A booth at the fair grounds has the shape of a hexagon and covers 375 square feet. An electrician estimates that the cost of

installing wiring around the outside of the booth will be 39.44 cents per foot. What will be the total cost of the wiring?

10. A water rheostat is often used for a load when testing generators. The rheostat consists of two iron terminals immersed in salt water and so arranged that one terminal can be moved toward or away from the other terminal. The movable terminal of a water rheostat has the shape of an equilateral triangle 32 inches on a side. It is .404 inch thick and is made of cast iron. What is its area on one side? How much does it weigh?

11. The wires between a generator and a switchboard are placed in a trough in the concrete floor. The trough is covered by sheet iron covers. One cover is triangular in shape having sides 8.4 inches, 6.8 inches and 10.2 inches. Find the area of this cover.

12. The most economical location of a generating station or a substation is at the load center of the system. Sometimes this location is found to be near the center of the business district in a city. In such locations the cost of land is excessive and the site available for a station is often of peculiar shape.

What is the area in square feet of a triangular piece of land having sides of 44 feet, 86 feet and 98 feet?

13. How many cubic feet of concrete are necessary to make 42 square concrete poles 31.1 foot high, having a butt 1 foot thick and a top 6 inches thick? (Frustum problem; see page 142.)

14. The mouthpiece of a telephone transmitter is made of nickel-plated material having a weight of .197 pound per cubic inch. It has the shape of the frustum of a cone. The volume of material in the mouthpiece is obtained by finding the difference between the frustums of cones having dimensions as follows:

	Large diameter	Small diameter	Altitude
	Ins.	Ins.	Ins.
Outside	$1\frac{3}{4}$	$1\frac{1}{16}$	$1\frac{1}{2}$
Inside	$1\frac{5}{8}$	1	$1\frac{1}{4}$

What is the weight of this mouthpiece?

15. A base for a small motor is made of wood weighing 45 pounds per cubic foot and has the shape of the frustum of a pyramid. It is hollow and its volume is obtained by finding the difference between two frustums of pyramids having dimensions as follows:

	Side of bottom	Side of top	Altitude
Outside	2 ft.	1 ft.	.5 ft.
Inside	1 ft. 9 in.	9 in.	.315 ft.

16. A copper funnel is made of two frustums of cones. Their dimensions are:

	Large diameter	Small diameter	Altitude
	Ins.	Ins.	Ins.
Large frustum	6	$\frac{3}{4}$	6
Small frustum	$\frac{3}{4}$	$\frac{1}{2}$	3

The copper is .117 inch thick. What is the weight of the funnel?

17. A planer which has a 6-inch pulley is driven by a motor having a 10-inch pulley. The distance between the centers of the pulleys is 7 feet. How long must the belt be, if an open belt is used? (See Fig. 154.)

18. A pulley with a diameter of 20 inches is connected by a crossed belt to another pulley with the same diameter. If the distance between the centers of the pulleys is 5 feet 8 inches what length of belt is required?

19. A crossed belt is used to belt a 9-inch pulley on a line shaft to an 18-inch pulley on a motor. If the center of the pulley is 4 feet 6 inches from the center of the shaft, what is the length of the belt?

20. What length of open belt is required to connect a 7-inch pulley on a line-shaft to a 15-inch pulley on a motor, if the distance between the centers of the pulleys is 6 feet?

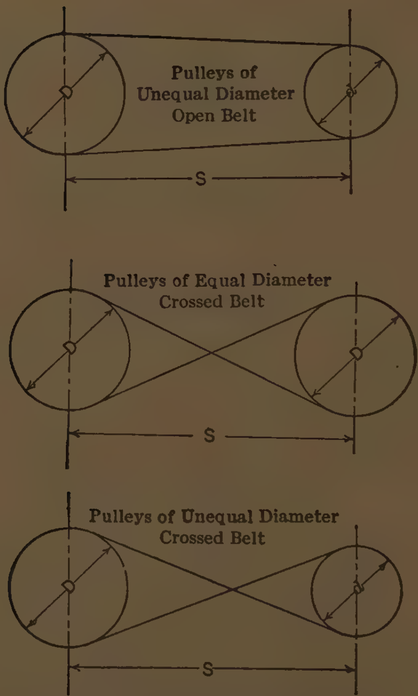


FIG. 154. Straight and Crossed Belts.

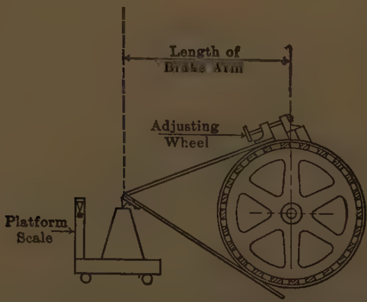


FIG. 155. Prony Brake for Brake Horse Power.

21. Figure 155 shows the device used to determine the brake horse power of an engine. The formula for finding this is as follows:

$$\text{b.h.p.} = \frac{4P \times R \times N}{21{,}000}.$$

In this formula b.h.p. stands for the brake horse power;

P stands for the weight on the scales at the end of the brake arm;

R stands for the length of the brake arm (this length must be reckoned in feet);

N stands for the number of revolutions per minute that the engine makes.

What is the brake horse power of an engine that makes 220 revolutions per minute if the length of the brake arm is 42 inches and the weight on the scales is 175 pounds?

INDEX

al